Actuator Fault and Link Failure Accommodation for Disturbed Multi-User Telerehabilitation Systems Subject to Communication Noise

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Abstract— **This paper investigates the Fault Tolerant Control (FTC) for Multi-User Telerehabilitation Systems (MUTSs) which are modelled as Itô stochastic differential equations with the occurrence of actuator loss-of-effectiveness faults, directed link failures, communication noise, and disturbances. An active FTC strategy using an Adaptive Sliding Mode Control (ASMC) method is addressed. By employing such strategy, MUTSs achieve stochastic consensus in the mean square sense onto the predefined stochastic switching sliding surfaces in finite time. Finally, Stochastic Input-to-State Stability (SISS) of the systems' states (positions, velocities, and orientations) will be shown and a numerical simulation will be provided.**

I. INTRODUCTION

Multilateral teleoperation systems have attracted interests of many researchers in the past decades. These systems have emerged to cope with some practical applications such as hazardous material transportation, space exploration etc. which are impossible by single-master, single-slave configuration. One of the main coordination/cooperation control schemes for multilateral teleoperation systems in the literature is the leader-follower control approach by which the states of slaves converge to the master's states using local information of their neighbours. In applications of multilateral teleoperation systems and particularly in telerehabilitation, the remote robots (slave robots) are long away from local robots (master robots). Thus, it is impossible to repair the distance robots at the time of failures.

Fault Tolerant Control (FTC) for a network of manipulators such as Multi-User Telerehabilitation Systems (MUTSs) enables the operator to teleoperate several distance robots and feel the force feedback in a reliable and safe way. Fault tolerability allows systems to overcome faults' effects and ensures acceptable performance for desired tasks mainly through control reconfigurations.

In this paper, we restrict our attention to the single-master multiple-slave configuration where the slave robots are teleoperated by a master and MUTSs are modelled as Multi-Agent Systems (MASs).

It should be pointed out that the performance of MASs in presence of actuator faults in some agents is investigated in the literature such as [1]–[7], to name a few. In addition, due to sensor imperfect measurements etc. communication noise is unavoidable. Also, randomly link failures may result in the failure or instability of the systems. Therefore, the proposed control approach is designed to address these challenges by employing Markovian jump structure. In recent years, Markovian Jump System (MJSs) have been extensively analyzed in networked systems ([8]–[12]).

The aim of this paper is to propose a Stochastic Input-to-State Stability (SISS) of MUTSs with considering communication noise, actuator faults, directed link failures, and disturbance simultaneously. To the best of our knowledge, there are no other publications in the literature that address the SISS of MUTSs. It is worth mentioning that this work is the first attempt to solve the active FTC of MUTSs by considering challenges mentioned before.

The remainder of the paper is organized into five sections. In the next section, mathematical background and notation are outlined. Next, system description will be provided. After that, the proposed controller laws which are based on the Adaptive Sliding Mode Control (ASMC) method will be discussed. In Section V, simulation results are given. Finally, the last section concludes the paper.

II. MATHEMATICAL BACKGROUND AND NOTATION

A. Notation

For a vector $w=(w_1,\cdots,w_n)$, $w_i\in\mathbb{R}$ and a matrix W, consider $\|w\|_2=\left(\sum_{i=1}^n w_i^2\right)^{\frac{1}{2}}$, $\|w\|_1=\left(\sum_{i=1}^n |w_i|\right)$ and $\|W\|_2=\lambda_{max}(W^*W)^{\frac{1}{2}}$ where $\lambda_{max}(\cdot)$ is the maximum eigenvalue of a matrix and W^* is the conjugate transpose of W. We define signum function as $\mathrm{sgn}(w)=[\mathrm{sgn}(w_1),\cdots,\mathrm{sgn}(w_n)]$ and diagonal matrix as $\mathrm{diag}(w)=\mathrm{diag}(w_1,\cdots,w_n)$. Let $\otimes$ denotes the Kronecker product, $\mathbf{1}=[1,1,\cdots,1]^T$, $\mathbf{0}=[0,0,\cdots,0]^T$ and I_m is an identity matrix. It should b noted that for convenience, we will write the $2-$norm as $\|\cdot\|$ to replace $\|\cdot\|_2$. Furthermore, a function $F(t)$ is said to be of class C^k if its derivatives for $k\in\{1,2,\cdots\}$ exist and are continuous (the continuity is implied by differentiability for all the derivatives except for $F^{(k)}(t)$).

The following definition describes the concept of the mean square stability of a random variable.

Definition 1 ([13]): Consider $r(t)$ to be a random variable. For a scalar $a>0$, $r(t)$ converges in the a'th mean to zero (or asymptotically a'th mean stable), if $\lim_{t\to\infty}E\{\|r(t)\|^a\}=0$. Also, we say that $r(t)$ is mean square stable, if $\lim_{t\to\infty}E\{\|r(t)\|^2\}=0$.

B. Graph Theory

Information exchange is modelled by a weighted digraph (or directed graph). Consider $G=(V,E,A)$ with the node set $V=\{V_1,\ldots V_N\}$, set of edges $E\subseteq V\times V$ and a weighted

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adjacency matrix $A(t) = [a_{ij}(t)]_{N\times N} \in \mathbb{R}^{N\times N}$, $a_{ij}(t) \geq 0$. $a_{ji}(t) > 0$ means that the i'th node receives information from node j then $e_{ij}(t) = (V_i, V_j) \in E$ and vice versa. $a_{ij}(t) = 0$ if $e_{ij} = (V_i, V_j) \notin E$ and the i'th node has no self-loop $(i = j)$. neighbours of the node i is defined as $N_i(t) = \{V_j \in V : (V_j, V_i) \in E\}$. A Sequence of edges in a directed graph of the form $e_{ij}, e_{jk}, \ldots$ is called a directed path. Graph G has at least one node with directed paths to all others nodes if and only if digraph G has a directed spanning tree. The Laplacian matrix $L(t) = [l_{ij}(t)]_{N\times N}$ of graph $G(t)$ is defined as $[l_{ij}(t)]_{N\times N}$ in which $l_{ij}(t) = -a_{ij}(t)$, $\forall\ i \neq j$ and $l_{ii}(t) = \sum_{j\in N_i(t), j\neq i} a_{ij}(t)$, $\forall i, j \in \{1, \ldots n\}$.

C. Markovian Process and Switching Topologies

In randomly switching topologies, MUTSs can be viewed as a class of stochastic MJSs in a complete fixed probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$ with filtration $\{\mathcal{F}_t\}_{t\geq 0}$ where Ω is the sample space, $\mathcal{F}$ is the σ-algebra of sample space subsets, and $\mathcal{P}$ is the probability measure on $\mathcal{F}$.

Consider a finite-state measurable Markovian process $\{\eta_t, t \in [0, \mathcal{T}]\}$ whose state-space is $\mathcal{S} \triangleq \{1, 2, \cdots, v\}$ and its generator is ψ_{ij} with transition probability p_{ij} from topology i at time t to topology j at the time $t + \delta$, $i, j \in \mathcal{S}$. Then, transition probability p_{ij} is given by

$$p_{i,j} = Prob(\eta_{t+\delta} = j | \eta_t = i) = \begin{cases} \psi_{i,j}\delta + o(\delta) & \text{if } i \neq j, \\ 1 + \psi_{i,i}\delta + o(\delta) & \text{if } n = j, \end{cases}$$

$$\psi_{i,i} = -\sum_{l=1, l\neq i}^{v} \psi_{il}, \psi_{il} \geq 0\ \forall i,\ j \in \mathcal{S},\ i \neq j.$$

where $\delta \geq 0$ and $\lim_{\delta\to 0, \delta>0} o(\delta)/\delta = 0$. The notation $o(\delta)$ denotes infinitesimal terms of order strictly higher than 1.

III. System Description, Orientation and Communication Noise model

A. System Description

In this paper, a network of MUTSs with Euler-Lagrange dynamics is considered such that each system (n-Degree Of Freedom (DOF) manipulator) is a node in the directed graph G. The dynamics of the manipulators can be expressed as

$$M(\theta_i)\ddot{\theta}_i + C(\theta_i, \dot{\theta}_i)\dot{\theta}_i + g(\theta_i) = \tau_i + \tau_{h_i} \quad (1)$$

in which τ_i is the controller input and τ_{h_i} is the force induced by an operator or environment.

From [14], the task space dynamic model of the master and slaves are given by the following equations, respectively.

$$M_l(\theta_l)\ddot{x}_l + C_l(\theta_l, \dot{\theta}_l)\dot{x}_l + g_l(\theta_l) = f_h + f_l \quad (2)$$

and

$$M_{ri}(\theta_{ri})\ddot{x}_{ri} + C_{ri}(\theta_{ri}, \dot{\theta}_{ri})\dot{x}_{ri} + g_{ri}(\theta_{ri}) = f_{ri} + f_{ei} \quad (3)$$

where $f_h := [h_h^T, m_h^T]^T$ and $f_{ei} := [h_{ei}^T, m_{ei}^T]^T$, $i \in \{1, \cdots, N\}$. x_l, x_{ri} represent task space trajectories of the master's and slaves' end-effectors, respectively. In addition, h_h, h_{ei} are the Cartesian linear forces and m_h, m_{ei} are Cartesian linear moments. Also, $M_{ri}(\theta_{ri})$, $M_l(\theta_l)$, $C_{ri}(\theta_{ri}, \dot{\theta}_{ri})$, and $C_l(\theta_l, \dot{\theta}_l)$ are $m \times m$, symmetric, positive-definite inertia and Coriolis centripetal matrices, respectively. In addition, $g_{ri}(\theta_{ri})$ and $g_l(\theta_l)$ are $m \times 1$ vectors of gravity force, f_{ri} and f_l are $m \times 1$ vectors of applied control forces, and f_h is an $m \times 1$ vector of external force acts on the master's end-effector by an operator. Also, f_{ei} is an $m \times 1$ vector of external force acts on the i'th slave's end-effector by its environment. Following this, reformulation of Equations (2) and (3) yields

$$\begin{aligned} \dot{x}_i &= v_i \\ \dot{v}_i &= \bar{h}_i(\theta_i, \dot{\theta}_i) + \bar{f}_i + \bar{f}_{ei},\ i \in \{0, 1, \cdots, N\} \end{aligned} \quad (4)$$

where $\bar{h}_i(\theta_i, \dot{\theta}_i) = -M_i^{-1}\Big(C_i(\theta_i, \dot{\theta}_i)v_i + g_i(\theta_i)\Big)$, $\bar{f}_{ei} = M_{ei}^{-1} f_{ei}$, and $\bar{f}_i = M_i^{-1} f_i$. Also, x_i and v_i are the task space trajectories and velocities of the end-effectors, respectively. We assume the agent 0 be the leader of MASs and the other agents $(\{1, \cdots, N\})$ are the followers. The following assumption ensures that the leader's states (positions, velocities, and orientations) are globally reachable from any i'th node.

Assumption 1: The graphs of MUTSs'networks are directed and weakly connected.

Remark 1: Consider a non-negative diagonal matrix $B = \text{diag}(\alpha_{10}, \cdots, \alpha_{N0}) \in \mathbb{R}^{N\times N}$ such that there exists at least a $\alpha_{i0} > 0$, $\forall i \in \{1, \cdots, N\}$ then $\tilde{L} = L + B$ is a full rank, symmetric, positive-definite matrix.

Assumption 2: $M_i(\cdot)$ is a positive-definite matrix. In addition, $M_i(\cdot)$, $C_i(\cdot)$, $g_i(\cdot)$ are C^1 functions. Also, f_{ei} is locally bounded.

B. Representing the orientation: Unit-quaternions

The unit-quaternion for the i'th robot is denoted by $\mathcal{J}_i$ which includes two elements $j_i \in \mathbb{R}$ and $J_i \in \mathbb{R}^3$ such that

$$\mathcal{J}_i = \begin{bmatrix} j_i \\ J_i \end{bmatrix} \in \mathbb{R}^4,\ j_i^2 + J_i^T J_i = 1 \quad (5)$$

From [15] and [16], the relation between the time-derivative of the unit-quaternion and the angular velocity is given by

$$\dot{\mathcal{J}}_i = \frac{1}{2}\mathcal{Z}^T(\mathcal{J}_i)\omega_i,\ \mathcal{Z}^T(\mathcal{J}_i) := \begin{bmatrix} -J_i^T \\ j_i I_3 - S(J_i) \end{bmatrix}$$

in which, $S(\cdot)$ is a skew-symmetric matrix operator. Therefore, augmented vector $\mathbf{x}_i$ are defined as

$$\dot{\mathbf{x}}_i = \begin{bmatrix} \dot{x}_i \\ \dot{\mathcal{J}}_i \end{bmatrix} = \Pi(\mathcal{J}_i)^T \begin{bmatrix} \dot{x}_i \\ \omega_i \end{bmatrix},\ \Pi(\mathcal{J}_i) = \begin{bmatrix} I_3 & \mathbf{0}_{3\times 4} \\ \mathbf{0}_3 & \frac{1}{2}\mathcal{Z}^T(\mathcal{J}_i) \end{bmatrix} \in \mathbb{R}^{6\times 7}$$

Assumption 3: There exists two constants ζ_1 and ζ_2 such that for states x_l, v_l, x_{ri} and v_{ri},

$$\Big\|\bar{h}_{ri}(\theta_{ri}, \dot{\theta}_{ri}) - \bar{h}_l(\theta_l, \dot{\theta}_l)\Big\| \leq \zeta_1\Big\|\mathbf{x}_{ri} - \mathbf{x}_l\Big\| + \zeta_2\Big\|v_{ri} - v_l\Big\|$$

C. Communication Noise Model

It is supposed that the i'th agent receives information from its neighbours thorough noisy channels which can be modelled for every node $j \in N_i(t)$ as $\mathbf{x}_{ji}^* = \mathbf{x}_{ji} + \sigma_{ji}\omega_{ji}$, $\mathbf{x}_{0i}^* = \mathbf{x}_{0i} + \sigma_{0i}\omega_{0i}$, $v_{ji}^* = v_{ji} + \sigma_{ji}\omega_{ji}$, and $v_{0i}^* = v_{0i} + \sigma_{0i}\omega_{0i}$ where $\Big\{\omega_{ji},\ i, j \in \{1, 2, \cdots, N\}\Big\}$ are independent standard white noises and σ_{ji} is the noise intensity. In addition, $\Sigma_i =$

$\mathrm{diag}\Big(\sigma_{1i}+\sigma_{10},\dots,\sigma_{Ni}+\sigma_{N0}\Big)$, $\Sigma=\mathrm{diag}\Big(\bar{\rho}_1^T\Sigma_1,\dots,\bar{\rho}_N^T\Sigma_N\Big)$, $\Sigma_{zi}=\mathrm{diag}\Big(\sigma_{1i},\dots,\sigma_{Ni}\Big)$, $\Sigma_z=\mathrm{diag}\Big(\bar{\alpha}_1^T\Sigma_{z1},\dots,\bar{\alpha}_N^T\Sigma_{zN}\Big)$, $\omega_i=\Big[\omega_{1i}+\omega_{10},\dots,\omega_{Ni}+\omega_{N0}\Big]^T$, and $\eta=\Big[\omega_1^T,\dots,\omega_N^T\Big]$. It has to be noted that $\bar{\rho}_i$ and $\bar{\alpha}_i$ refer to i'th row of $\tilde{L}$ and L, respectively.

IV. Main Results

A. System Reformulation

Consider Equation (4), each robot's consensus error can be defined as

$$\begin{aligned} e_{\mathbf{x}_i}&=\textstyle\sum_{j=1}^N a_{ij}(t)(\mathbf{x}_{ri}-\mathbf{x}_{rj})+\alpha_{i0}(t)(\mathbf{x}_{ri}-\mathbf{x}_l)\\ e_{v_i}&=\textstyle\sum_{j=1}^N a_{ij}(t)(v_{ri}-v_{rj})+\alpha_{i0}(t)(v_{ri}-v_0)\end{aligned} \tag{6}$$

Equation (6) in a collective form with considering the MJSs are reformulated as

$$\begin{aligned} \dot{\Lambda}_1&=\Big(\tilde{L}(\eta_t)\otimes I_m\Big)\underline{\Pi}(\mathcal{J})(\dot{X}_N-\dot{X}_r)\\ \dot{\Lambda}_2&=\Big(\tilde{L}(\eta_t)\otimes I_m\Big)\Big(\bar{H}-1\otimes\bar{h}_0+\bar{F}-1\otimes\bar{f}_0\Big)\end{aligned} \tag{7}$$

where $X_N=\Big[\mathbf{x}_1^T,\dots,\mathbf{x}_N^T\Big]^T$, $X_r=1\otimes\mathbf{x}_0$, $\underline{\Pi}(\mathcal{J})=\Big[\Pi(\mathcal{J}_i)^T,\cdots,\Pi(\mathcal{J}_N)^T\Big]^T$, $\bar{H}=\Big[\bar{h}_1(\theta_1,\dot{\theta}_1),\dots,\bar{h}_N(\theta_N,\dot{\theta}_N)\Big]^T$, $\bar{h}_0=\bar{h}_0(\theta_0,\dot{\theta}_0)$, $\bar{F}=\Big[\bar{f}_1^T+\bar{f}_{e1}^T,\dots,\bar{f}_N^T+\bar{f}_{eN}^T\Big]^T$, $\bar{F}_E=\Big[\bar{f}_{e1}^T,\dots,\bar{f}_{eN}^T\Big]^T$, $\bar{f}_0=\bar{f}_0^T+\bar{f}_{0e}^T$, $\Lambda_1=\Big[e_{\mathbf{x}1}^T,e_{\mathbf{x}2}^T,\dots,e_{\mathbf{x}N}^T\Big]^T$, and $\Lambda_2=\Big[e_{v1}^T,e_{v2}^T,\dots,e_{vN}^T\Big]^T$.

Remark 2: The Leader-following stochastic consensus of MUTSs is equivalent to the SISS of System (7).

B. Actuator Fault Model: Loss of Effectiveness Faults

In System (7), actuator faults are modelled in the following collective form $\bar{F}=(I_{Nm}-\mathcal{Q})\bar{F}^{\mathcal{D}}$ and distributed form of i'th actuator fault is,

$$\bar{f}_i=(I_m-\mathcal{Q}_i)\bar{f}_i^{\mathcal{D}} \tag{8}$$

where $\bar{f}_i^{\mathcal{D}}$ denotes the desired control input. In addition, $\mathcal{Q}=\mathrm{diag}(\mathcal{Q}_1,\cdots,\mathcal{Q}_N)$ and $\mathcal{Q}_i=\mathrm{diag}(q_{i1},q_{i2},\cdots,q_{im})$ represent the fault severity of the actuators in which the scalar q_{ij} satisfying $0\le\underline{q}_{ij}\le q_{ij}\le\bar{q}_{ij}<1$, $\forall i\in\{1,\cdots,N\}$ and $j\in\{1,\cdots,m\}$. The scalar q_{ij} is called as the Effectiveness Loss Value (ELV) of the j'th actuator of i'th slave.

Remark 3: The actuators' fault model (8) covers the fault-free cases ($\underline{q}_{ij}=\bar{q}_{ij}=0$) and the faulty cases ($0<\underline{q}_{ij}\le q_{ij}\le\bar{q}_{ij}<1$). However, failure cases ($\underline{q}_i=\bar{q}_{ij}=1$) are excluded in (8) since the matrix $(I-\mathcal{Q})$ will not be a full rank matrix which is necessary in the controller design.

C. Reachability of Stochastic Sliding Surfaces

Through the online estimation of ELVs of faulty actuators, it is shown that controller laws (13) to (15) can drive the system's dynamic onto the sliding surfaces (9).

Theorem 1: Consider System (7), fault model (8), Assumptions 1, 2, and 3. The distributed control protocols (13) to (15) with the control input $\bar{f}_i^{\mathcal{D}}=f_{ai}+f_{bi}+f_{ci}$, $i\in\{0,1,\cdots,N\}$ and updating law (11) make the sliding surfaces

$$\Gamma=\Lambda_2+\Upsilon_1\Lambda_1+\Upsilon_2\int_0^t\Lambda_1(\tau)d\tau-\Upsilon_3\int_0^t F_E(\tau)d\tau \tag{9}$$

finite time mean square stable if for a $\Xi(k)$, $k\in\mathcal{S}$, Linear Matrix Inequalities (LMIs)

$$-\Omega(k)\Xi(k)-\Xi^T(k)\Omega(k)+\sum_{j=1}^{\nu}\psi_{kj}\Omega(j)\prec 0 \tag{10}$$

has symmetric, positive-definite matrix solutions $\Omega(k)$, $k\in\mathcal{S}$.

Proof: See Appendix 1. ■

The updating law for $\hat{q}_{ij}$, $i\in\{1,\cdots,N\}$ and $j\in\{1,\cdots,m\}$ is given as

$$\dot{\hat{q}}_{ij}=\begin{cases}0 & \{\ \hat{q}_{ij}=\underline{q}_{ij}\text{ and }\chi\le 0\}\text{ or }\{\hat{q}_{ij}=\bar{q}_{ij}\text{ and }\chi\ge 0\}\\ \chi & \text{Otherwise.}\end{cases} \tag{11}$$

where $\chi=\mathrm{s}_{ij}\Gamma^T\Omega_{ii}\Big(\tilde{L}^{-1}\otimes I_m\Big)_{c:i}\Big(1-\hat{q}_{ij}\Big)^{-1}\Big(F_a+F_b\Big)_{r:i}$. $\mathrm{s}_{ij}>0$ is the updating gain and $\mathcal{A}_{c:i}$ and $\mathcal{B}_{r:i}$ refer i'th column and row of $\mathcal{A}$ and $\mathcal{B}$, respectively.

Remark 4: Updating law (11) projects the estimate $\hat{q}_{ij}$ into the interval $[\underline{q}_{ij},\bar{q}_{ij}]$. It is assumed that in the actuator fault model (8), the lower and upper bounds $\underline{q}_{ij},\bar{q}_{ij}$ satisfy $0<\underline{q}_{ij}\le q_{ij}\le\bar{q}_{ij}<1$. This means that $0<1-\hat{q}_{ij}\le 1$. Hence, $I-\hat{\mathcal{Q}}$ is invertible.

D. Sliding Motion Analysis

From now on, we examine SISS of the sliding surfaces (9).

Definition 2: For System (9), a function $V(x,t)\in\mathscr{C}^{2,1}(\mathbb{R}^n\times[t_0,\infty);\mathbb{R}_+)$ is called an SISS-Lyapunov function, if there exists functions $\bar{\alpha},\underline{\alpha}\in\mathscr{K}_\infty,\alpha,\chi\in\mathscr{K}$ such that for all $x\in\mathbb{R}^n,u\in\mathbb{R}^m$ and $t\ge t_0$ and finite-state measurable Markovian process $\{\eta_t,t\in[0,\mathcal{T}]\}$ provided that $\underline{\alpha}(|x|)\le V(x,t,\eta_t)\le\bar{\alpha}(|x|)$ and for each possible value $\eta_t=k$, $k\in\mathcal{S}$:

$$\begin{aligned}\mathscr{L}V(x,t,k)&=\frac{\partial V(x,t,k)}{\partial t}+\frac{\partial V(x,t,k)}{\partial x}f(t,x,k,u)\\ &+\frac{1}{2}Tr\Big(g^T(t,x,k,u)\frac{\partial^2 V(x,t,k)}{\partial x^2}g(x,t,k,u)\Big)\\ &+\textstyle\sum_{j=1}^{\nu}\gamma_{kj}x\le-\alpha(|x|),\ \forall|x|\ge\chi(\|u\|)\end{aligned}$$

where $\mathscr{L}$ is the infinitesimal generator.

Theorem 2: System (9) is SISS if there exists an SISS-Lyapunov function $(V;\alpha_1,\alpha_2,\alpha,\chi)$ and the function $\alpha\circ\alpha_2^{-1}$ is convex.

Proof: This theorem is the extension of Theorem 2 in [17], thus it is skipped for brevity. ■

The following theorem ensures SISS of the sliding surfaces (9).

Theorem 3: The sliding surfaces (9) is stochastic input to state stable, if for given positive-definite matrices $N(k)$, there exist positive-definite matrices $P(k)$ satisfying

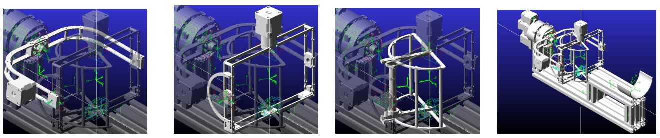
$$2P(k)Z+\sum_{j=1}^{\nu}\psi_{kj}P(j)+N(K)=0,\ k\in S \tag{12}$$

where $Z = \begin{bmatrix} 0 & 1 \\ -\Upsilon_2 & -\Upsilon_1 \end{bmatrix}$.

Proof: The proof follows from [1, Theorem 4.5] (skipped due to the page limit). ■

V. Simulation Example

As a numerical example, we consider MUTSs of five AUTWRIST rehabilitation robots (depicted in Fig. 1). The kinematic and dynamic model of this robot is in the form of (1) and was introduced in [18, Appendix A] which satisfies Assumption 2.



(a) (b) (c) (d)

Fig. 1: AUTWRIST Robot (a) First Rotation ψ. (b) Second Rotation θ. (c) Third Rotation ϕ. (d) ADAMS Model.

The task space dynamic model (4) has the following well-known property ([14]) that for all θ, $\dot{\theta}$ and $\dot{x}$, there exists a constant $\kappa \in \mathbb{R}^+$ such that the right hand side of the equation $C(\theta,\dot{\theta})\dot{x} \leq \kappa|\dot{x}|^2$ is absolutely continuous then one concludes its Lipschitz continuity. Therefore, Assumption 3 is satisfied.

The slaves interact with their environments while Cartesian linear moments f_{e_i}, $i \in \{1,\cdots,N\}$ as disturbances act on the end-effectors (constrained motion) during the times $t = 32s$ and $t = 40s$. This force for each slave may be described by $f_{e_i} = -K_e(\mathbf{x}_i - \mathbf{x_0}) - B_e(\dot{\mathbf{x}}_i - \dot{\mathbf{x}}_\mathbf{0}) + 0.5 \times \sin(t) \times \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ where $\mathbf{x_0}$ and $\dot{\mathbf{x}}_\mathbf{0}$ are desired trajectory and velocity in the Cartesian space and $K_e = 5 \times I_2$ and $B_e = I_2$. This force is depicted in Fig. 2(a). According to Assumption 2, f_{e_i} is locally bounded.

It is assumed that at $t = 2s$, the actuator of joint 1 in slave 3, the actuator of joint 2 in slave 4 and the actuator of joint 2 in slave 2 are faulty with ELVs $q_{31} = 0.5$, $q_{42} = 0.6$ and $q_{22} = 0.1$, respectively. Also, in fault model (8), we assume $\underline{q}_{ij} = 0$ and $\bar{q}_{ij} = 0.75$ for all slaves and their actuators. In addition, let the intensity of the measurement noise $\sigma_{ji} = 0.1$, $\forall i, j = \{0,1,\cdots,N\}$.

The network topologies are depicted in Fig. 3. From Fig. 3, it can be seen that there is a spanning tree in all topologies which means that Assumption 1 is satisfied. Also, the switching signal for MJSs is depicted in Fig. 2(b) and the corresponding transition probability matrix is given in [1, Equation (45)].

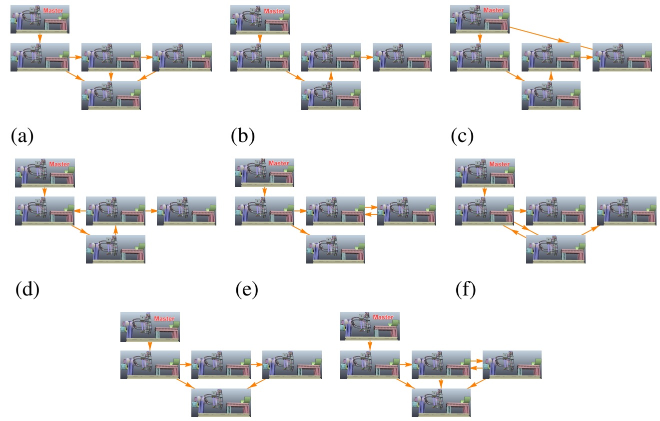


Fig. 3: Communication topologies.

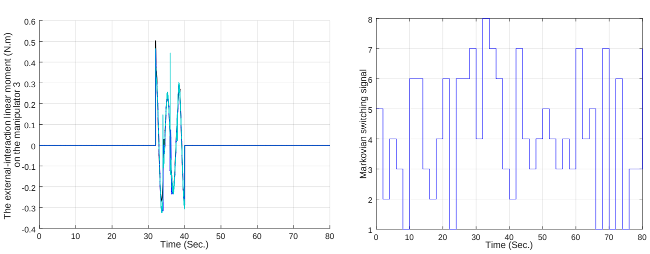
The forms of the desired sliding surfaces and the proposed distributed controller are as (9) and (13) to (15), respectively with the following parameters $\Upsilon_1 = M_d^{-1}C_d$, $\Upsilon_2 = M_d^{-1}K_d$, and $\Upsilon_3 = M_d^{-1}$ with $M_d = 0.2 \times I_3$, $K_d = I_3$, and $C_d = 1.7 \times I_3$. Also, the feasibility of the LMIs (10) and Equation (12) has been verified in [1, Section 5]

Orientation of the first joint in slave 3 compared to the corresponding master's orientation is shown in Fig. 5. The initial positions and velocities are chosen randomly. Due to the page limit, comparison results for other joints and slaves are omitted. Also, estimation of the fault severity of the faulty actuators using updating law (11) are shown in Fig. 6. The control inputs applied to the joints of slave 3 and the corresponding sliding surfaces are depicted in Fig. 4(a) and Fig. 4(b), respectively.

Remark 5: It is worth to mention that from Fig. 6, the estimated values $\hat{q}_{42}$, $\hat{q}_{31}$ and $\hat{q}_{22}$ can converge but not necessarily to their true values (here, $q_{42} = 0.6$, $q_{31} = 0.5$ and $q_{22} = 0.1$).

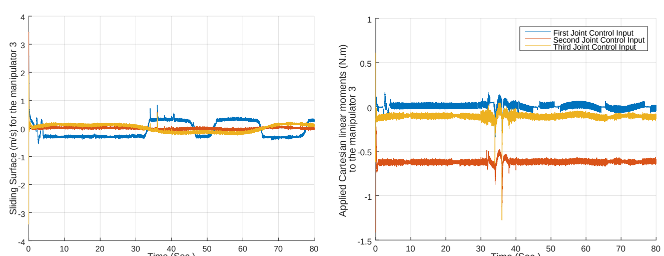
VI. Conclusion

In this paper, the focus is on the fault tolerant coordinated control for MUTSs with the directed link failures



(a) (b)

Fig. 2: (a): The external linear moment (b):Markovian Switching Signal



(a) (b)

Fig. 4: (a): Stochastic Sliding Surfaces of Slave 3 (b):Control Inputs applied to the joints of Slave 3.

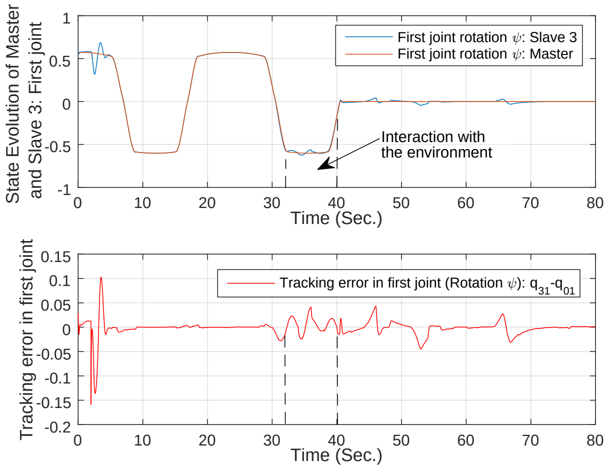


Fig. 5: rotation ψ

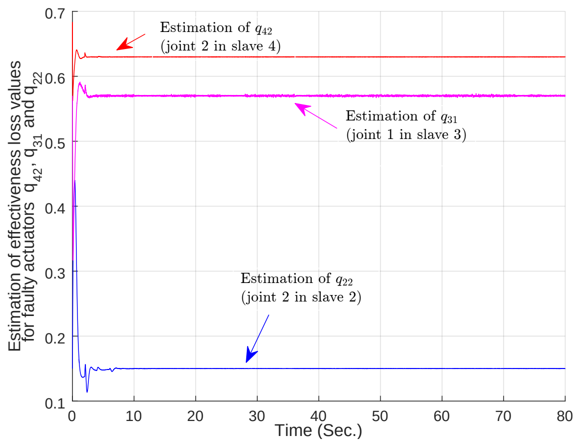


Fig. 6: Estimation of ELVs for faulty actuators: $\hat{q}_{42}$, $\hat{q}_{31}$ and $\hat{q}_{22}$.

and communication noise in the presence of the actuator faults and disturbances. We address a definition for stochastic input-to-state stable Lyapunov function and two theorems for assurance of SISS based on the ASMC method and weak infinitesimal operation in term of LMIs. The simulation results illustrate the validity of the presented algorithms. In the terms of future work, communication time-delays and sensor faults may be considered.

Appendix 1: Proofs

Proof: [Proof of Theorem 1] Consider the desired distributed protocol law $\bar{f}_i^{\mathcal{D}} = f_{ai} + f_{bi} + f_{ci}$ where

$$f_{ai} = \frac{1}{d_i+\alpha_{i0}}(I_m - \hat{\mathbb{Q}}_i)^{-1}\Big(\sum\nolimits_{j=1}^N a_{ij}(\eta_t) f_{aj}^* + \alpha_{i0} f_0^*\Big) \quad (13)$$

$$\begin{aligned} f_{bi} = {} & \frac{1}{d_i+\alpha_{i0}}\Big(I_m - \hat{\mathbb{Q}}_i\Big)^{-1}\Big[\sum\nolimits_{j=1}^N a_{ij}(\eta_t) f_{bj}^* \\ & -\Upsilon_{1ii}\Big(\sum\nolimits_{j=1}^N a_{ij}(\eta_t)(v_i - v_j^*) + \alpha_{i0}(v_i - v_0^*)\Big) \\ & -\Upsilon_{2ii}\Big(\sum\nolimits_{j=1}^N a_{ij}(\eta_t)(\mathbf{x}_i - \mathbf{x}_j^*) + \alpha_{i0}(\mathbf{x}_i - \mathbf{x}_0^*)\Big) + \Upsilon_{3ii}\bar{f}_{ie}^T\Big] \end{aligned} \quad (14)$$

$$\begin{aligned} f_{ci} = {} & \frac{1}{d_i+\alpha_{i0}}\Big[\sum_{j=1}^N a_{ij}(\eta_t) f_{cj}^* + \xi_{ii}(\eta_t)\Big(\sum_{j=1}^N a_{ij}(\eta_t)(v_i - v_j^*) \\ & + \alpha_{i0}(v_i - v_0^*) + \Upsilon_{1ii}\Big(\sum\nolimits_{j=1}^N a_{ij}(\eta_t)(\mathbf{x}_i - \mathbf{x}_j^*) + \alpha_{i0}(\mathbf{x}_i - \mathbf{x}_0^*)\Big) \\ & + \Upsilon_{2ii}\int_0^t \Big(\sum\nolimits_{j=1}^N a_{ij}(\eta_t)(\mathbf{x}_i - \mathbf{x}_j^*) + \alpha_{i0}(\mathbf{x}_i - \mathbf{x}_0^*)\Big) d\tau \\ & - \Upsilon_{3ii}\int_0^t \bar{f}_{ie}^T d\tau\Big) - \Phi^A sgn\Big(\rho_{ii}\Big(\sum\nolimits_{j=1}^N a_{ij}(\eta_t)(v_i - v_j^*) \\ & + \alpha_{i0}(v_i - v_0^*) + \Upsilon_{1ii}\Big(\sum_{j=1}^N a_{ij}(\eta_t)(\mathbf{x}_i - \mathbf{x}_j^*) + \alpha_{i0}(\mathbf{x}_i - \mathbf{x}_0^*)\Big) \\ & + \Upsilon_{2ii}\int_0^t \Big(\sum_{j=1}^N a_{ij}(\eta_t)(\mathbf{x}_i - \mathbf{x}_j^*) + \alpha_{i0}(\mathbf{x}_i - \mathbf{x}_0^*)\Big) d\tau \\ & - \Upsilon_{3ii}\int_0^t \bar{f}_{ie}^T d\tau\Big)\Big)\Big] \end{aligned} \quad (15)$$

in which $f_0^* = f_0 + \sigma_{0i}\omega_{0i}$, $f_{aj}^* = f_{aj} + \sigma_{ji}\omega_{ji}$, $f_{bj}^* = f_{bj} + \sigma_{ji}\omega_{ji}$, and $f_{cj}^* = f_{cj} + \sigma_{ji}\omega_{ji}$. In addition, $\hat{\mathbb{Q}}_i = \text{diag}\Big\{\hat{q}_{i1}, \hat{q}_{i2}, \cdots, \hat{q}_{im}\Big\}$ where $\hat{q}_{ij}$ is the estimation of ELV for the j'th actuator of i'th slave. In f_{ci}, Φ^A is defined as

$$\begin{aligned} \Phi^A = {} & \frac{1}{1-\bar{q}_{max}}\Big[\Big\| -(\tilde{L}(k)^{-1}\otimes I_m)(1\otimes f_0)\Big\| + \omega\Big\|\phi\otimes f_0\Big\| \\ & + c_0 + g + \Big\|\Upsilon_1\dot{\Lambda}_1 + \Upsilon_2\Lambda_1 - \Upsilon_3 F_E\Big\| + \omega\Big\|\Upsilon_3 F_E - \Upsilon_1\dot{\Lambda}_1 - \Upsilon_2\Lambda_1\Big\|\Big] \end{aligned}$$

in which c_0 is a small positive number, $g = \omega(\zeta_1\|\Lambda_1\| + \zeta_2\|\Lambda_2\|)$, $\omega = \|\tilde{L}(k)\|\|\tilde{L}(k)^{-1}\|$. Also, Υ_{1ii}, Υ_{2ii}, Υ_{3ii}, ξ_{ii} and ρ_{ii} are diagonal elements of diagonal matrices Υ_1, Υ_2, Υ_3, Ξ, and Ω, respectively. Collective form of control protocol (13) to (15) can be written as

$$\begin{aligned} F_a &= \Big(I - \hat{\mathbb{Q}}\Big)^{-1}\Big(\tilde{L}(\eta_t)^{-1}\otimes I_m\Big)\Big[(\phi\otimes f_0) + \Sigma\eta\Big] \\ F_b &= \Big(I - \hat{\mathbb{Q}}\Big)^{-1}\Big(\tilde{L}(\eta_t)^{-1}\otimes I_m\Big)\Big[-\Upsilon_1\dot{\Lambda}_1 - \Upsilon_2\Lambda_1 + \Upsilon_3 F_E \\ &\qquad - \Upsilon_1\Sigma\eta - \Upsilon_2\Sigma\eta + \Sigma_z\eta\Big] \\ F_c &= -\Big(\tilde{L}(\eta_t)^{-1}\otimes I_m\Big)\Big[\Xi(\eta_t)\Gamma + \Phi^A sgn\Big(\Omega(\eta_t)\Gamma\Big) + \Sigma_z\eta\Big] \end{aligned} \quad (16)$$

where $\phi = [\alpha_{10}, \ldots, \alpha_{N0}]^T$ and $\hat{\mathbb{Q}} = \text{diag}\Big\{\hat{\mathbb{Q}}_1, \cdots, \hat{\mathbb{Q}}_N\Big\}$.

Now, Consider the Lyapunov function

$$V(\Gamma,\eta_t,t)=\Gamma^T\Omega(\eta_t)\Gamma+\sum_{i=1}^{N}\sum_{j=1}^{N}\mathrm{S}_{ij}^{-1}\tilde{q}_{ij}^2 \tag{17}$$

where $\tilde{q}_{ij}=\hat{q}_{ij}-q_{ij}$ is the estimation error and $\tilde{\mathcal{Q}}_i=\mathrm{diag}\Big\{\tilde{q}_{i1},\cdots,\tilde{q}_{im}\Big\},\ \forall i\in\{1,2,\cdots,N\}$.

For each possible value $\eta_t=k,\ k\in\mathcal{S}$, weak infinitesimal of the Lyapunov function (17) by using binomial inverse $\Big(I-\hat{\mathcal{Q}}\Big)^{-1}=I+\Big(I-\hat{\mathcal{Q}}\Big)^{-1}\hat{\mathcal{Q}}$ yields

$$\begin{aligned}\mathcal{L}V(\Gamma,k,t)&=\Gamma^T\Omega(k)\dot{\Gamma}+\dot{\Gamma}^T\Omega(k)\Gamma+\Gamma^T\Big(\sum_{j=1}^{\nu}\psi_{kj}\Omega(j)\Big)\Gamma\\&+2\sum_{i=1}^{N}\sum_{j=1}^{N}\mathrm{S}_{ij}^{-1}\tilde{q}_{ij}\dot{\tilde{q}}_{ij}+\frac{1}{2}Tr\Big(\Psi_0^T(\Omega(k)+\Omega^T(k))\Psi_0\Big)\end{aligned}$$

where $\Psi_0=\Big(\tilde{L}(\eta_t)\otimes I_m\Big)(I-\mathcal{Q})\Big(\tilde{L}(\eta_t)^{-1}\otimes I_m\Big)\Big(\Sigma+2\Sigma_z\Big)$.

Therefore, by using the differential form of sliding surfaces (9) which is

$$d\Gamma=d\Lambda_2+\Upsilon_1 d\Lambda_1+\Upsilon_2\Lambda_1 dt-\Upsilon_3 F_E dt$$

one concludes that

$$\begin{aligned}\mathcal{L}V(\Gamma,k,t)&\le 2\Gamma^T\Omega(k)\Big[-(\tilde{L}(k)^{-1}\otimes I_m)(1\otimes f_0)\\&+\Upsilon_1\dot{\Lambda}_1+\Upsilon_2\Lambda_1-\Upsilon_3F_E+\big(\tilde{L}(k)^{-1}\otimes I_m\big)\big(F_a+F_b\big)\\&+\big(\tilde{L}(k)^{-1}\otimes I_m\big)(\tilde{\mathcal{Q}})(I-\hat{\mathcal{Q}})^{-1}(F_a+F_b)\\&+(\tilde{L}(k)^{-1}\otimes I_m)\Big[\bar{H}-1\otimes h_0+(I-\mathcal{Q})F_c\Big]\Big]\\&+\frac{1}{2}Tr\Big(\Psi_0^T(\Omega(k)+\Omega^T(k))\Psi_0\Big)+2\sum_{i=1}^{N}\sum_{j=1}^{N}\mathrm{S}_{ij}^{-1}\tilde{q}_{ij}\dot{\tilde{q}}_{ij}\end{aligned} \tag{18}$$

From (10) and with substituting (16) into (18), we conclude that

$$\begin{aligned}\mathcal{L}V(\Gamma,k,t)&\le 2\Big\|\Gamma\Omega(k)\Big\|\Big[\Big\|-(\tilde{L}(k)^{-1}\otimes I_m)(1\otimes f_0)\Big\|\\&+\Big\|\Upsilon_1\dot{\Lambda}_1+\Upsilon_2\Lambda_1-\Upsilon_3F_E\Big\|+\Big\|\phi\otimes f_0\Big\|\\&+\Big\|-\Upsilon_1\dot{\Lambda}_1-\Upsilon_2\Lambda_1+\Upsilon_3F_E\Big\|\Big]+2\sum_{i=1}^{N}\sum_{j=1}^{N}\mathrm{S}_{ij}^{-1}\tilde{q}_{ij}\dot{\tilde{q}}_{ij}\\&+2\Gamma\Omega(k)\big(\tilde{L}(k)^{-1}\otimes I_m\big)(\tilde{\mathcal{Q}})(I-\hat{\mathcal{Q}})^{-1}(F_a+F_b)\\&-2\Phi^A(1-\bar{q}_{max})\Big\|\Gamma\Omega(k)\Big\|_1+2g\Big\|\Gamma\Omega(k)\Big\|\\&+\frac{1}{2}Tr\Big((\Psi_0)^T(\Omega(k)+\Omega^T(k))(\Psi_0)\Big)\end{aligned}$$

Note that considering the updating law (11), one has

$$\begin{aligned}&2\Gamma\Omega(k)\Big(\tilde{L}(k)^{-1}\otimes I_m\Big)(\tilde{\mathcal{Q}})(I-\hat{\mathcal{Q}})^{-1}\Big(F_a+F_b\Big)\\&\qquad+2\sum_{i=1}^{N}\sum_{j=1}^{m}\mathrm{S}_{ij}^{-1}\tilde{q}_{ij}\dot{\tilde{q}}_{ij}\le 0\end{aligned}$$

From now on, the rest of the proos is the same as the proof of Theorem 4.1 in [1].

■

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