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Fault-Tolerant Consensus of Nonlinear Multi-Agent Systems with Directed Link Failures, Communication Noise and Actuator Faults

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ABSTRACT
This paper deals with the problem of Fault-Tolerant Control (FTC) for a consensus of nonlinear Multi-Agent Systems (MASs) with directed link failures/recoveries in the presence of communication noise and actuator faults. The directed link failures/recoveries model considered here is randomly switching topologies. Randomly switching topologies in MASs governed by Markovian jump process. Moreover, the actuator loss-of-effectiveness faults may happen in any of the actuators. To deal with this, by using SMC method and weak infinitesimal operation, a passive FTC strategy is presented and sufficient conditions for stochastic consensus of underlying MASs are derived in the mean square stability sense. To that end, by employing such FTC design, MASs achieve consensus in mean square sense onto the predefined stochastic sliding surfaces in finite time while robustness of the overall system against the directed link failures/recoveries and communication noise and actuator faults is guaranteed. Finally, an example on multiple networked Euler-Lagrange systems is presented to verify the effectiveness and feasibility of the proposed algorithms.

KEYWORDS
Fault tolerant control, stochastic consensus, multi-agent systems, link failures, communication noise, actuator faults, switching topologies, Markovian jump process, weak infinitesimal operation

1. Introduction

Fault tolerability, good redundancy, great robustness, and reliability are attractive topics and essential aspects of practical applications in the consensus of MASs that have aroused interests of many researchers recently.

Consensus means that all agents' states reach a predefined or undefined agreement using local information of their neighbors. Consensus problems can be categorized into leaderless consensus and leader-following consensus. Leaderless consensus protocols try to make an agreement between the states of all agents on a common value which depends on initial states of agents using local information of each agent's neighbors. Despite this, leader-following consensus protocols make that all states of followers converge to a leader's states.

In realistic environments, a consensus of MASs in a network are specified by agents' intrinsic dynamics and their interactions that can be affected by many uncertain factors

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including limited communication bandwidth or sensing ranges, network-induced time delays, random packet drops, undesirable external disturbances, failure of physical devices, and so on. Also, because of sensors' imperfect measurements, electromagnetic interference, quantization errors etc., communication noise is unavoidable. Therefore, the authors assume that the information is exchanged between the agents in a network through noisy communication. In other words, it is supposed that agents receive local inaccurate measurements from their neighbors. Another key point to remember is that with noise, all agents' states are convergent in a stochastic sense to a random variable rather than a deterministic value and stochastic properties of a random variable such as the expectation and the variance should be considered.

Link failures may result in serious degradation of performance indices of the system and lead to instability and even failure of the system's operation. A large class of MASs are subjected to variable but predefined communication topologies with the occurrence of link failures/recoveries which can be modeled as dynamically switching topologies. Moreover, dynamically switching topologies can be considered as a special case of random switching structures. Due to the reasons mentioned before, in realistic situations, communication topologies are considered as randomly switching topologies which seems to be a standard model in the analyzing of MASs with link failures/recoveries. To that end, Markovian chain process is used to characterize the random link failures/recoveries between the agents.

Fault tolerant control protocol should be able to compensate effects of the faults or attenuate them through reconfiguration of the controller algorithms. Fault tolerant systems ensure acceptable performance and desired safety and reliability. The controller architectures should be able to attenuate undesired effects which caused by actuator faults and guarantee the acceptable performance of the closed-loop system.

The following literature gives reviews of a consensus of MASs with dynamically and randomly switching topologies, communication noise and actuator fault with special attention to results reported in recent years.

Wen in (Wen, Duan, Chen, & Yu, 2014) solves consensus problem of MASs with Lipschitz-type node dynamics and dynamically switching topologies, if each possible topology contains a directed spanning tree. In (Z. Wang, Xi, Yao, & Liu, 2015) sufficient condition for cost consensus was shown that only dependent on the maximum eigenvalue of the Laplacian matrices of dynamically switching undirected topologies.

Reference (Wen, Yu, Peng, & Rahmani, 2016) is investigated consensus tracking problem for second-order nonlinear inherent dynamical MASs with switching topologies and a time-varying reference state. In (K. Chen, Wang, & Zhang, 2016) a consensus of first-order nonlinear MASs under switching directed topologies is considered. By designing a suitable switching law, MASs can reach consensus if the switching topologies jointly contain a spanning tree.

The authors in (Liu, Zhou, Qi, & Wu, 2016) have considered leaderless consensus problem of MASs with Lipschitz nonlinearities under switching directed topology with two assumptions on communication topology. First assumption is that each possible topology contains a directed spanning tree and other assumption is strongly connectedness and balancedness of underlying topology.

Regarding stochastic features in switching patterns due to the probabilistic nature of communication channels failures, considering randomly switching topologies and noisy communications would be challenging and meaningful. Moreover, dynamically switching topologies can be considered as special cases of randomly switching topologies. Therefore, considering randomly switching topologies is a challenging problem in the study of MASs' communication topologies.

A stochastic asymptotically consensus protocol for MASs with Markovian jump in directed topologies, linear dynamic and external disturbances is studied in (Luan, Zhou, Ding, & Liu, 2015). Lei Ding (Ding, Han, Guo, & Zhang, 2014) addresses consensus problem for a nonlinear first-order system under Markovian switching topologies and network-induced delay.

Kim et al. (Kim, Park, & Choi, 2014) investigated mean square consensus for Leaderless and leader-following MASs which are described by first-order and second-order dynamics with random link failures between each agent while random link failures are presented by a Bernoulli probability sequence.

Shang in (Shang, 2014) addressed consensus in general linear MASs over randomly switching directed topology with Markovian process switching law. If union of the topologies corresponding to a positive recurrent states of the Markov process possesses an acyclic partition then the consensus can be reached regardless of the couplings strength among agents. Also, Shang et al. (Shang, 2015) showed couple-group consensus of general linear time-invariant dynamical MASs with Markovian switching topologies. By the linear consensus protocol, agents in one sub-network reach an agreement while the other ones in other sub-network reach another agreement.Necessary and sufficient condition is derived such that mean square consensus can be achieved if and only if a union of the underlying graphs has a spanning tree in closed sets.

In a recent paper by Shang (Shang, 2016), the authors studied stochastic consensus for linear time-invariant MASs over Markovian switching directed networks with topology's uncertainties and time-varying delays. If a union of topologies corresponding to positive recurrent states of the Markov process has a spanning tree and each agent's dynamic is stabilizable then consensus can be reached.

Mean square consensus of a discrete-time linear time-invariant MASs with communication noise studied in (Y. Wang, Cheng, Hou, et al., 2015) by using a time-varying consensus gain to decrease noise effect. The consensus is concentrated under different directed topology's conditions. Firstly, if the communication topology has a spanning tree and each node has at least one parent node (leaderless case). Secondly, if the communication topology has a spanning tree and there exists one node without any parent node (leader-following case). Thirdly, if the communication topology does not have a spanning tree. Reference (Y. Wang, Cheng, Ren, Hou, & Tan, 2015) addressed a stochastic consensus problem of linear MASs with communication noise and Markovian switching directed topologies under some linear control protocols and derived sufficient conditions for the mean square and almost sure consensus.

The mean square consensus of linear continuous-time linear time-invariant dynamical MASs with communication noise with fixed directed communication topology is studied in (Cheng, Hou, & Tan, 2014). The Consensus conditions are derived for three cases. Firstly, the communication topology has a spanning tree and every node has at least one parent node. Secondly, the communication topology has a spanning tree and there exists one node without any parent node. Finally, communication topology has no spanning tree.

Reference (Shen, Shi, Zhu, & Zhang, 2017) has considered the leader-following consensus with the time-varying transmission nonlinearities. Adaptive distributed controllers are proposed to compensate nonlinearities and solve the consensus problem.

The authors in (Shen, Shi, & Shi, 2016; Shi & Shen, 2017) studied the leader-following consensus problem of uncertain high-order nonlinear MASs. It is assumed that only the relative outputs are available. observer-based distributed adaptive control and distributed adaptive fuzzy control schemes are proposed to guarantee that all followers asymptotically reach a consensus on a leader's states, respectively. Also, references (Shen & Shi, 2016; Shen et al., 2017) addressed adaptive output consensus of uncertain nonlinear MASs with saturation and unknown nonlinear dead-zone, respectively.

It should be pointed out that the performance of MASs in the presence of actuator faults in some agents is investigated in the literature such as (S. Chen, Ho, Li, & Liu, 2015; Saboori & Khorasani, 2015; Semprun, Yan, Butt, & Chen, 2017; Xiao, Yin, & Gao, 2017; Ye, Zhao, & Cao, 2016; Zhou, Li, & Liu, 2017; Zhu & Yang, 2016), to name a few.

Designing a consensus protocol which addresses a fault tolerant consensus of nonlinear MASs with directed link failures/recoveries in the presence of communication noise and actuator fault simultaneously is an important challenging problem. In contrast with the reported papers and to the best of the authors' knowledge, results on the FTC of a stochastic consensus of nonlinear MASs with directed link failures, communication noise, and actuator faults have not been reported and this technical note is the first attempt in considering nonlinearity of MASs, communication noise, directed link failure, and actuator faults simultaneously. The conclusion of this note not only contributes to the theory development but also applicable to many the practical problems.

By employing SMC method and weak infinitesimal operation, sufficient conditions for the mean square consensus of underlying MASs are derived. To that end, by employing a passive FTC strategy, MASs achieve mean square consensus onto the predefined sliding surfaces in finite time while robustness of the overall system against the directed link failures/recoveries, communication noise and actuator faults is guaranteed and the specific information of the model uncertainties and bounds of nonlinear terms are unknown in the controller design process.

Firstly, the consensus of nonlinear MASs on predefined stochastic sliding surfaces with directed link failures, communication noise and actuator faults is transformed to a stochastic input-to-state stability problem of a nonlinear randomly switched systems. It should be noted that switching law is described by Markovian jump process. Secondly, it is shown that the mean square consensus can be reached, if there exists a spanning tree in each possible topology.

The rest of the paper is organized as follows. In section 2, mathematical framework and notation will be discussed. Section 3 addresses the systems description and some assumptions. Main results on the stochastic input-to-state stability of the sliding surfaces are introduced in section 4. Next, section 5 is included a numerical example to demonstrate the effectiveness of the theoretical results obtained. Finally, section 6 summarizes the results of this work and draws conclusions and future work.

2. Mathematical Background and Notation

2.1. Notation

Consider a vector $\vartheta = (\vartheta_1, \dots \vartheta_N)$, $\vartheta_i \in \mathbb{R}$ and a matrix Φ, norm p of the vector is $\|\vartheta\|_p = (\sum_{i=1}^{n} |\vartheta_i|^p)^{\frac{1}{p}}$ for the real number $p \geq 1$ and norm 2 of the matrix Φ is $\|\Phi\|_2 = \lambda_{\max}(\Phi^*\Phi)^{\frac{1}{2}}$ where $\lambda_{\max}(\cdot)$ denote maximum eigenvalue of a matrix. Also, $\wp_{max}(\Phi)$ and $\wp_{min}(\Phi)$ denote maximum and minimum singular values of Φ, respectively. Let $\mathbf{1}_n = (1, 1, \dots, 1)^T \in \mathbb{R}^n$, $\mathbf{0}_n = (0, 0, \dots, 0)^T \in \mathbb{R}^n$. I_n denotes $n \times n$ identity matrix. For a random variable/vector β, $\mathbb{E}[\beta]$ is used to represent the mathematical expectation. $\bigotimes$ denotes the Kronecker product. The inner product of two vectors will be denoted by

$\langle \cdot\ ,\ \cdot \rangle$. Also, $\operatorname{diag}(\cdot)$ represents a diagonal matrix formed by its entries. By $\lim_{\delta\downarrow 0,\delta>0}$, we denote to the one-sided limit $\lim_{\delta\to 0,\delta>0}$. If a function $\psi : \mathbb{R}^+ \to \mathbb{R}^+$ is continuous, strictly increasing and $\psi(0) = 0$ then it is called a class $\mathcal{K}$ function. Besides, if it satisfies $\psi(u) \to \infty$ as $u \to \infty$, then it is defined a class $\mathcal{K}_\infty$ function. A continuous function $\beta : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ belongs to a class $\mathcal{KL}$ function, if for any fixed $t \in \mathbb{R}^+$, the function $\beta(., t)$ belongs to a class $\mathcal{K}$ function and for any fixed $s \in \mathbb{R}^+$, the function $\beta(s, .)$ is decrescent and $\beta(s, .) \to 0$ as $s \to \infty$. A function $F(t)$ is said to be of (differentiability) class C^k if its derivatives for $k \in \{1, 2, ...\}$ exist and are continuous. The continuity is implied by differentiability for all the derivatives except for $F^{(k)}(t)$.

Definition 2.1. (Billingsley, 2013; Higham, 2000) Consider $r(t)$ as a random variable. For a real number $a \geq 1$, $r(t)$ converges in the a'th mean (or in the L^a-norm) towards zero, if $E[\|r(t)\|^a]$ exists, and $\lim_{t\to\infty} E[\|r(t)\|^a] = 0$. Also, we say that $r(t)$ is mean square stable if $\lim_{t\to\infty} E[\|r(t)\|^2] = 0$.

2.2. Graph Theory

In this work, the leader-following MASs consisting of $N + 1$ agents is considered. The agents are labeled as $\{0, 1, \cdots, N\}$. We assume the agent 0 be the leader of MAS and the other agents $\{1, \cdots, N\}$ are the followers.

In MASs, the information exchanged among the agents can be modeled by an interaction weighted digraph (or directed graph). Consider $G = (V, E, A)$ with the node set $V = \{V_1, \ldots V_N\}$, set of edges $E \subseteq V \times V$, and a weighted adjacency matrix $A(t) = [a_{ij}(t)]_{N\times N} \in \mathbb{R}^{N\times N}$, $a_{ij}(t) \geq 0$. $a_{ji}(t) > 0$ means that the i'th agent receives information from the agent j then $e_{ij}(t) = (V_i, V_j) \in E$ and vice versa. $a_{ij}(t) = 0$, if $e_{ij} = (V_i, V_j) \notin E$ and the i'th node has no self-loop ($i = j$) in the graph. The neighbors of the node i are defined as $N_i(t) = \{V_j \in V : (V_j, V_i) \in E\}$. A Sequence of edges in a directed graph of the form $e_{ij}, e_{jk}, \ldots$ is called directed path. A directed tree is referred to directed graph in which every node has exactly one parent except root node such that it has directed path to all other nodes. Graph G has at least one node with directed paths to all others nodes if and only if digraph G has a directed spanning tree. The Laplacian matrix $L(t) = [l_{ij}(t)]_{N\times N}$ of graph $G(t)$ is defined as $[l_{ij}(t)]_{N\times N} : l_{ij}(t) = -a_{ij}(t)\ \forall i \neq j$, $l_{ii}(t) = \sum_{j\in N_i(t), j\neq i} a_{ij}(t)$ $\forall i, j \in \{1, \ldots n\}$. Also, $L(t) = D(t) - A(t)$ in which $D(t) = \operatorname{diag}[d_1(t), \ldots d_n(t)]$ where $d_i(t) = \sum_{j\in N_i(t)} a_{ij}(t)$ for each $i, j \in \{1, \ldots n\}$. The following assumption ensures that the leader's states (positions and velocities) are globally reachable from any i'th node.

Assumption 2.2. *The communication topologies are directed and weakly connected.*

Remark 1. Consider a non-negative diagonal matrix $B := \operatorname{diag}(\alpha_{10}(t), \ldots, \alpha_{N0}(t)) \in \mathbb{R}^{N\times N}$ such that there exists at least a $\alpha_{i0}(t) > 0$, $\forall i \in \{1, \cdots, N\}$ then the matrix $\tilde{L} = L + B$ is a full rank symmetric positive definite matrix and Assumption 2.2 holds.

2.3. Markovian Process and Switching Topologies

As mentioned before, the authors consider randomly switching topologies to model directed link failures/recoveries. In randomly switching topologies, MASs can be viewed as a class of stochastic systems with Markovian jump structures in a complete fixed probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$ with filtration $\{\mathcal{F}_t\}_{t\geq 0}$, where Ω is the sample space, $\mathcal{F}$ is the σ-algebra of the sample space subsets, and $\mathcal{P}$ is the probability measure on $\mathcal{F}$.

Finite-state measurable Markovian process $\{\eta_t, t \in [0, \mathcal{T}]\}$ whose state-space is $\mathcal{S} \triangleq \{1, 2, \cdots, v\}$ and its generator (ψ_{ij}) with transition probability (p_{ij}) from the topology i at time t to the topology j at time $t + \delta$, $i, j \in \mathcal{S}$ is given by

$$
\begin{aligned}
p_{ij} &= Prob(\eta_{t+\delta} = j | \eta_t = i) \\
&= \begin{cases} \psi_{i,j}\delta + o(\delta) & \text{if } i \neq j \\ 1 + \psi_{i,i}\delta + o(\delta) & \text{if } i = j \end{cases} \\
\psi_{i,i} &= -\sum_{l=1, l\neq i}^{v} \psi_{il}, \psi_{il} \geq 0 \forall i, l \in \mathcal{S}, i \neq l
\end{aligned}
\tag{1}
$$

where $\delta \geq 0$ and $\lim_{\delta\to 0, \delta>0} o(\delta)/\delta = 0$. Indeed, The notation $o(\delta)$ denotes infinitesimal terms of order strictly higher than one in δ.

3. Systems Description

Multi-agent systems with Euler-Lagrange dynamics are considered here and the consensus problem with time varying state reference and directed link failures/recoveries in the presence of communication noise and actuator faults is addressed.

The dynamics of the leader may be described by the following equation

$$
M_0(\theta_0)\ddot{x}_0 + C_0(\theta_0, \dot{\theta}_0)\dot{x}_0 + g_0(\theta_0) = f_0 + f_{0e},\ x_0 \in \mathbb{R}^m,\ \dot{x}_0 = v_0 \in \mathbb{R}^m,\ t \geq 0. \tag{2}
$$

the follower's dynamics are described as follow

$$
M_i(\theta_i)\ddot{x}_i + C_i(\theta_i, \dot{\theta}_i)\dot{x}_i + g_i(\theta_i) = f_i + f_{ie},\ x_i \in \mathbb{R}^m,\ \dot{x}_i = v_i \in \mathbb{R}^m,\ t \geq 0. \tag{3}
$$

in which, for any $i \in \{1, \cdots, N\}$, $M_i(\theta_i)$ is a $m \times m$ symmetric positive definite matrix and $C_i(\theta_i, \dot{\theta}_i)$ is a $m \times m$ Coriolis centripetal matrix. Also, $G_i(\theta_i)$ is a $m \times 1$ vector of gravity force. f_i and f_{ie} are a $m \times 1$ vectors of applied generalized force and external force acting on the end-effector of the robots. In addition, x_i and v_i denote the Cartesian position and velocity of i'th end-effector. Following this, equations (2) and (3) are reformed as

$$
\begin{aligned}
\dot{x}_i &= v_i, \\
\dot{v}_i &= \bar{h}_i(\theta_i, \dot{\theta}_i) + \bar{f}_i + \bar{f}_{ie},\ i = 0, 1, \cdots, N
\end{aligned}
\tag{4}
$$

where $\bar{h}_i(\theta_i, \dot{\theta}_i) = -M_i^{-1}\Big(C_i(\theta_i, \dot{\theta}_i)v_i + g_i(\theta_i)\Big)$, $\bar{f}_i = M_i^{-1} f_i$, and $\bar{f}_{ie} = M_i^{-1} f_{ie}$.

Assumption 3.1. *M_i , C_i , g_i are C^1 functions on states of each agent $[x_i, v_i]$. Also, $\bar{f}_{ei}$ is locally bounded.*

Assumption 3.2. *For any x_i, v_i, $i \in \{0, 1, \cdots, N\}$ and $t \geq 0$ there exist two constants*

ζ_1 *and* ζ_2 *such that*

$$\left\| \bar{h}_i(\theta_i, \dot{\theta}_i) - \bar{h}_0(\theta_0, \dot{\theta}_0) \right\| \leq \zeta_1 \|x_i - x_0\| + \zeta_2 \|v_i - v_0\| \tag{5}$$

The task space dynamic model (4) has the following well-known property (Siciliano, Sciavicco, Villani, & Oriolo, 2010; Spong, Hutchinson, Vidyasagar, et al., 2006) that for all θ_i, $\dot{\theta}_i$ and $\dot{x}_i$ there exists a constant $\kappa_i \in \mathbb{R}^+$, $i \in \{0, 1, \cdots, N\}$ such that

$$C(\theta_i, \dot{\theta}_i)\dot{x}_i \leq \kappa |\dot{x}_i|^2, \;\; i \in \{0, 1, \cdots, N\} \tag{6}$$

Since right hand of equation (6) is absolutely continuous, one can conclude its Lipschitz continuity. Therefore, Assumption 3.2 is satisfied.

4. Main Results

4.1. System Reformulation

Define each agent's consensus error as following

$$\begin{aligned} e_{x_i} &= \sum\nolimits_{j=1}^{N} a_{ij}(t)(x_i - x_j) + \alpha_{i0}(t)(x_i - x_0) \\ e_{v_i} &= \sum\nolimits_{j=1}^{N} a_{ij}(t)(v_i - v_j) + \alpha_{i0}(t)(v_i - v_0) \end{aligned}$$

where $i \in \{1, \cdots, N\}$.

We transformed above equations in the following collective form as a class of stochastic systems with Markovian jump process η_t with the parameters in the probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathcal{P})$ and the filtration $\{\mathcal{F}_t\}_{t \geq 0}$

$$\begin{aligned} \dot{\Lambda}_1 &= (\tilde{L}(t) \otimes I_m)(\dot{X}_N - \dot{X}_r) \\ \dot{\Lambda}_2 &= (\tilde{L}(t) \otimes I_m)(\ddot{X}_N - \ddot{X}_r) = (\tilde{L}(\eta_t) \otimes I_m)(\bar{H} - 1 \otimes \bar{h}_0 + \bar{F} - 1 \otimes \bar{f}_0) \end{aligned} \tag{7}$$

where $X_N = [x_1^T, \ldots, x_N^T]^T \in \mathbb{R}^{m \times N}$, $X_r = 1 \otimes x_0$, $\bar{H} = [\bar{h}_1^T(\theta_1, \dot{\theta}_1), \ldots, \bar{h}_N^T(\theta_N, \dot{\theta}_N)]^T$, $\bar{h}_0 = \bar{h}_0^T(\theta_0, \dot{\theta}_0)$, $\bar{F} = [\bar{f}_1^T + \bar{f}_{1e}^T, \ldots, \bar{f}_N^T + \bar{f}_{Ne}^T]^T$, $\bar{F}_E = [\bar{f}_{1e}^T, \ldots, \bar{f}_{Ne}^T]^T$, $\bar{f}_0 = \bar{f}_0^T + \bar{f}_{0e}^T$, $\Lambda_1 = [e_{x1}^T, e_{x2}^T, \ldots, e_{xN}^T]^T$, and $\Lambda_2 = [e_{v1}^T, e_{v2}^T, \ldots, e_{vN}^T]^T$.

4.2. Actuator Fault Model: Loss-of-Effectiveness Faults

In system (7), it is assumed that the actuator faults may happen which is modeled in the following distributed form

$$\bar{f}_i = (I_m - \mathcal{Q}_i)\bar{f}_i^{\mathcal{D}} \tag{8}$$

where $\bar{f}_i^{\mathcal{D}}$ denotes the desired control input. The collective form of actuator faults is $\bar{F} = (I_{Nm} - \mathcal{Q})\bar{F}^{\mathcal{D}}$ in which $\mathcal{Q} = \text{diag}(\mathcal{Q}_1, \cdots, \mathcal{Q}_N)$ and $\mathcal{Q}_i = \text{diag}(q_{i1}, q_{i2}, \cdots, q_{im})$ which represents the fault severity of i'th follower and j'th actuator for each agent. It is assumed that the scalar q_{ij} satisfying $0 \leq \underline{q}_{ij} \leq q_{ij} \leq \bar{q}_{ij} < 1$, $i = \{1, 2, \cdots, N\}$ and

$j = \{1, 2, \cdots, m\}$. In other words, the scalar q_{ij} is usually termed as the effectiveness loss value of the j'th actuator of the i'th follower.

Remark 2. It is worthwhile to mention that fault model (8) covers the fault-free case ($\underline{q}_{ij} = \bar{q}_{ij} = 0$) and the faulty case ($0 < \underline{q}_{ij} \leq q_{ij} \leq \bar{q}_{ij} < 1$). Since $(\tilde{L}(t) \otimes I_m)(I - \mathcal{Q})$ should be a full rank matrix then the complete failure cases ($\underline{q}_{ij} = \bar{q}_{ij} = 1$) are excluded in (8).

4.3. Communication Noise Model

It is assumed that the i'th agent receives the information from its neighbors thorough noisy channels. For every agent $j \in N_i(t)$, noisy information are modeled as $x^*_{ji} = x_{ji} + \sigma_{ji}\omega_{ji}$ $v^*_{ji} = v_{ji} + \sigma_{ji}\omega_{ji}$, $x^*_{0i} = x_{0i} + \sigma_{0i}\omega_{0i}$, and $v^*_{0i} = v_{0i} + \sigma_{0i}\omega_{0i}$ where $\{\omega_{ji}; i, j \in \{1, \cdots, N\}\}$ are independent standard white noises and σ_{ji} is the noise intensity. In addition, we define $\varSigma_i = \mathrm{diag}(\sigma_{1i} + \sigma_{10}, \ldots, \sigma_{Ni} + \sigma_{N0})$, $\varSigma = \mathrm{diag}(\rho_1^T \varSigma_1, \ldots, \rho_N^T \varSigma_N)$, $\varSigma_{zi} = \mathrm{diag}(\sigma_{1i}, \ldots, \sigma_{Ni})$, $\varSigma_z = \mathrm{diag}(\alpha_1^T \varSigma_{z1}, \ldots, \alpha_N^T \varSigma_{zN})$, $\omega_i = [\omega_{1i} + \omega_{10}, \ldots, \omega_{Ni} + \omega_{N0}]^T$, and $\eta = [\omega_1^T, \ldots, \omega_N^T]$. It has to be noted that ρ_i and α_i denotes i'th row of matrix $\tilde{L} = L + B$ and L, respectively.

4.4. Reachability of Stochastic Sliding Surfaces

In this section, it will be proved that the controllers (11) to (13) not just make the sliding surfaces (9) globally attractive but also stable in mean square sense in finite time.

Theorem 4.1. *Consider Assumptions* (2.2), (3.1), *and* (3.2). *The distributed control protocols* (11) *to* (13) *with the control input* $f_i^{\mathcal{D}} = f_{ai} + f_{bi} + f_{ci}$ *and fault model* (8) *make the sliding surfaces*

$$\varGamma = \varLambda_2 + \varUpsilon_1(\eta_t)\varLambda_1 + \varUpsilon_2(\eta_t)\int_0^t \varLambda_1(\tau)d\tau - \varUpsilon_3(\eta_t)\int_0^t F_E(\tau)d\tau \tag{9}$$

stable in mean square sense in finite time for system (4) *and any possible structure* $\eta_t = k$, $k \in \mathcal{S}$, *if there exist* $\Theta(k)$, $k \in \mathcal{S}$ *such that the following inequalities*

$$-(1 - \underline{q}_{min})\Omega(k)\Theta(k) - (1 - \underline{q}_{min})\Theta^T(k)\Omega(k) + \sum\nolimits_{j=1}^{\nu} \psi_{kj}\Omega(j) \prec 0 \tag{10}$$

have symmetric positive-definite matrix solutions $\Omega(k)$.

$$f_{ai} = (d_i + \alpha_{i0})^{-1} \sum\nolimits_{j=1}^{N} (a_{ij}(\eta_t) f^*_{aj} + \alpha_{i0} f^*_0) \tag{11}$$

$$
\begin{aligned}
f_{bi} = (d_i+\alpha_{i0})^{-1}\Bigg[&\sum\nolimits_{j=1}^{N} a_{ij}(\eta_t)f^*_{bj} - \Upsilon_{1ii}(\eta_t)\Bigg(\sum\nolimits_{j=1}^{N} a_{ij}(\eta_t)(v_i-v^*_j)+\alpha_{i0}(v_i-v^*_0)\Bigg)\\
&-\Upsilon_{2ii}(\eta_t)\Bigg(\sum\nolimits_{j=1}^{N} a_{ij}(\eta_t)(x_i-x^*_j)+\alpha_{i0}(x_i-x^*_0)\Bigg)+\Upsilon_{3ii}(\eta_t)\bar{f}^T_{ie}\Bigg]
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
f_{ci} = (d_i+\alpha_{i0})^{-1}\Bigg[&\sum\nolimits_{j=1}^{N} a_{ij}(\eta_t)f^*_{cj} + \theta_{ii}(\eta_t)\Bigg(\sum\nolimits_{j=1}^{N} a_{ij}(\eta_t)(v_i-v^*_j)+\alpha_{i0}(v_i-v^*_0)+\\
&+\Upsilon_{1ii}(\eta_t)\Big(\sum\nolimits_{j=1}^{N} a_{ij}(\eta_t)(x_i-x^*_j)+\alpha_{i0}(x_i-x^*_0)\Big)\\
&+\Upsilon_{2ii}(\eta_t)\int_0^t\Big(\sum\nolimits_{j=1}^{N} a_{ij}(\eta_t)(x_i-x^*_j)+\alpha_{i0}(x_i-x^*_0)\Big)d\tau-\Upsilon_{3ii}(\eta_t)\int_0^t \bar{f}^T_{ie}(\tau)d\tau\Bigg)\\
&-\mathit{\Phi}_i\ sgn\Bigg(\varrho_{ii}\Bigg(\sum\nolimits_{j=1}^{N} a_{ij}(\eta_t)(v_i-v^*_j)+\alpha_{i0}(v_i-v^*_0)\\
&+\Upsilon_{1ii}(\eta_t)\Big(\sum\nolimits_{j=1}^{N} a_{ij}(\eta_t)(x_i-x^*_j)+\alpha_{i0}(x_i-x^*_0)\Big)\\
&+\Upsilon_{2ii}(\eta_t)\int_0^t\Big(\sum\nolimits_{j=1}^{N} a_{ij}(\eta_t)(x_i-x^*_j)+\alpha_{i0}(x_i-x^*_0)\Big)d\tau-\Upsilon_{3ii}(\eta_t)\int_0^t \bar{f}^T_{ie}(\tau)d\tau\Bigg)\Bigg)\Bigg]
\end{aligned}
\tag{13}
$$

in which Υ_{1ii}, Υ_{2ii}, Υ_{3ii}, θ_{ii}, and ϱ_{ii} are positive diagonal elements of diagonal matrices Υ_1, Υ_2, Υ_3, Θ, and Ω in each switching mode, respectively. Also,

$$
\begin{aligned}
\mathit{\Phi}_i = \frac{1}{\omega\big(1-\bar{q}_{max}\big)}\Bigg[&c_0(\eta_t)+g+\Big\|-f_0\Big\|+\Big\|\Upsilon_{1ii}(\eta_t)\Bigg(\sum\nolimits_{j=1}^{N} a_{ij}(\eta_t)(v_i-v^*_j)\\
&+\alpha_{i0}(v_i-v^*_0)\Bigg)+\Upsilon_{2ii}(\eta_t)\Bigg(\sum\nolimits_{j=1}^{N} a_{ij}(\eta_t)(x_i-x^*_j)+\alpha_{i0}(x_i-x^*_0)\Bigg)-\Upsilon_{3ii}(\eta_t)\bar{f}^T_{ie}\Big\|\\
&+(1-\underline{q}_{min})\omega\Big\|\alpha_{i0}f_0\Big\|+(1-\underline{q}_{min})\omega\Big\|-\Upsilon_{1ii}(\eta_t)\Bigg(\sum\nolimits_{j=1}^{N} a_{ij}(\eta_t)(v_i-v^*_j)\\
&+\alpha_{i0}(v_i-v^*_0)\Bigg)-\Upsilon_{2ii}(\eta_t)\Bigg(\sum\nolimits_{j=1}^{N} a_{ij}(\eta_t)(x_i-x^*_j)+\alpha_{i0}(x_i-x^*_0)\Bigg)+\Upsilon_{3ii}(\eta_t)\bar{f}^T_{ie}\Big\|\Bigg]
\end{aligned}
\tag{14}
$$

where c_0 is a small positive number, $g = \omega(\zeta_1\|\Lambda_1\| + \zeta_2\|\Lambda_2\|)$, and $\omega = \big\|\tilde{L}(k)\big\|\big\|\tilde{L}(k)^{-1}\big\|$. According to the notion in section 4.3, $f^*_{ai} = f_{ai} + \sigma_{ji}\omega_{ji}$, $f^*_0 = f_0 + \sigma_{0i}\omega_{0i}$, $f^*_{bi} = f_{bi} + \sigma_{ji}\omega_{ji}$, $f^*_{ci} = f_{ci} + \sigma_{ji}\omega_{ji}$, and $i, j \in \{1, \cdots, N\}$.

Proof. First of all, we obtain the collective forms of the control protocols (11) to (13).

Using equation (11), f_{ai} can be written as following:

$$f_{ai} = (d_i + \alpha_{i0})^{-1}\Big(\sum_{j=1}^{N} a_{ij}(\eta_{lt}) f_{aj} + \alpha_{i0} f_0\Big) + (d_i + \alpha_{i0})^{-1}\Big(\sum_{j=1}^{N} a_{ij}(\eta_{lt})\sigma_{ij}\omega_{ij} + \alpha_{i0}\sigma_{i0}\omega_{i0}\Big) \tag{15}$$

in collective form, equation (15) can be written such as:

$$F_a = \Big(\tilde{L}(\eta_{lt})^{-1} \otimes I_m\Big)\Big[(\phi \otimes f_0) + \Sigma\eta\Big] \tag{16}$$

where $\phi = [\alpha_{10}, \ldots, \alpha_{N0}]^T$.

In the same way, we separate noisy terms of (12) and (13). At the end, we obtain collective forms of f_{bi}, f_{ci}, and Φ_i as following

$$F_b = \Big(\tilde{L}(\eta_{lt})^{-1} \otimes I_m\Big)\Big[-\Upsilon_1(\eta_{lt})\dot{\Lambda}_1 - \Upsilon_2(\eta_{lt})\Lambda_1 + \Upsilon_3(\eta_{lt})F_E - \Upsilon_1(\eta_{lt})\Sigma\eta - \Upsilon_2(\eta_{lt})\Sigma\eta + \Sigma_z\eta\Big] \tag{17}$$

Moreover,

$$F_c = \Big(-\tilde{L}(\eta_{lt})^{-1} \otimes I_m\Big)\Bigg[\Theta(\eta_{lt})\ \Gamma + \Phi\ sgn\Bigg(\Omega(\eta_{lt})\Gamma\Bigg) + \Sigma_z\eta\Bigg] \tag{18}$$

and

$$\Phi = \frac{1}{\omega\big(1-\bar{q}_{max}\big)}\Bigg[c_0(\eta_{lt}) + g + \big\| -(\tilde{L}(\eta_{lt}) \otimes I_m)(1 \otimes f_0)\big\| + \big\|\Upsilon_1(\eta_{lt})\dot{\Lambda}_1 + \Upsilon_2(\eta_{lt})\Lambda_1 - \Upsilon_3(\eta_{lt})F_E\big\| + (1-\underline{q}_{min})\omega\big\|\phi \otimes f_0\big\| + (1-\underline{q}_{min})\omega\big\| -\Upsilon_1(\eta_{lt})\dot{\Lambda}_1 - \Upsilon_2(\eta_{lt})\Lambda_1 + \Upsilon_3(\eta_{lt})F_E\big\|\Bigg] \tag{19}$$

It should be noted that following distributed protocol laws $\bar{f}_i^{\mathcal{D}} = f_{ai} + f_{bi} + f_{ci}$, $i \in \{0, 1, \cdots, N\}$. can be written as $\bar{F}^{\mathcal{D}} = F_a + F_b + F_c$.

Now we consider the Lyapunov candidate function as

$$V(\Gamma, \eta_{lt}, t) = \Gamma^T \Omega(\eta_{lt})\Gamma \tag{20}$$

and the following typical system

$$dx(t) = F(x(t), t, \eta_{lt})dt + G(x(t), t, \eta_{lt})dw(t) \tag{21}$$

Let $C^{2,1}(\mathbb{R}^n \times \mathbb{R}_+ \times S; \mathbb{R}_+)$ denote the family of all non-negative functions $V(\Gamma, t, \eta_{lt})$ on $\mathbb{R}^n \times \mathbb{R}_+ \times S$ which are twice continuously differentiable in x and once differentiable in t. If $V \in C^{2,1}(R^n \times R_+ \times S; R_+)$, define an operator $\mathcal{L}V$ from $\mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}_+ \times S$

on $\mathbb{R}$ by (22) for the typical system (21):

$$\mathcal{L}V(x,t,i) = V_t(x,t,i) + V_x(x,t,i)F(x,t,i) + \frac{1}{2}trace\Big[G^T(x,t,i)V_{xx}(x,t,i)G(x,t,i)\Big] + \sum\nolimits_{j=1}^{\nu} \psi_{ij} V(x,t,j) \tag{22}$$

where

$$\begin{aligned} V_t(x,t,i) &= \frac{\partial V(x,t,i)}{\partial t},\ V_x(x,t,i) = \left(\frac{\partial V(x,t,i)}{\partial x_1}, \dots, \frac{\partial V(x,t,i)}{\partial x_n}\right) \\ V_{xx}(x,t,i) &= \left(\frac{\partial^2 V(x,t,i)}{\partial x_i \partial x_j}\right)_{n\times n} \end{aligned} \tag{23}$$

Differential of the sliding surfaces (9) yields

$$d\mathit{\Gamma} = d\mathit{\Lambda}_2 + \mathit{\Upsilon}_1(\eta_t)d\mathit{\Lambda}_1 + \mathit{\Upsilon}_2(\eta_t)\mathit{\Lambda}_1 dt - \mathit{\Upsilon}_3(\eta_t)F_E dt \tag{24}$$

We separate deterministic and stochastic terms of (24) to reach the standard format of the typical system (21). Since $\eta(t)dt = dw(t)$, the factor of $dw(t)$ in the typical system (21) can be written as

$$\Psi_0 = \Big(\tilde{L}(\eta_t) \otimes I_m\Big)(I - \mathcal{Q})\Big(\tilde{L}(\eta_t)^{-1} \otimes I_m\Big)\Big(\mathit{\Sigma} + 2\mathit{\Sigma}_z\Big) \tag{25}$$

For each possible value $\eta_t = k$, $k \in \mathcal{S}$, weak infinitesimal of the Lyapunov function (20) yields

$$\begin{aligned} \mathcal{L}V(\mathit{\Gamma},k,t) = {} & \mathit{\Gamma}^T \mathit{\Omega}(k)\dot{\mathit{\Gamma}} + \dot{\mathit{\Gamma}}^T \mathit{\Omega}(k)\mathit{\Gamma} + \mathit{\Gamma}^T\Big(\sum\nolimits_{j=1}^{\nu} \psi_{kj}\mathit{\Omega}(j)\Big)\mathit{\Gamma} \\ & + \frac{1}{2}trace\left[\Psi_0^T\Big(\mathit{\Omega}(k) + \mathit{\Omega}^T(k)\Big)\Psi_0\right] \end{aligned} \tag{26}$$

Substituting equations (16)-(18) into equation (26) and using LMI (10) yield

$$\begin{aligned}
\mathcal{L}V(\mathit{\Gamma},k,t) \leq{}& 2\mathit{\Gamma}^T\mathit{\Omega}(k)\bigg[-(\tilde{L}(k)\otimes I_m)(1\otimes f_0)+\mathit{\Upsilon}_1(\eta_t)\dot{\mathit{\Lambda}}_1+\mathit{\Upsilon}_2(\eta_t)\mathit{\Lambda}_1-\mathit{\Upsilon}_3(\eta_t)F_E\\
&+(\tilde{L}(k)\otimes I_m)(I-\mathfrak{Q})F_a+(\tilde{L}(k)\otimes I_m)(I-\mathfrak{Q})F_b\\
&+(\tilde{L}(k)\otimes I_m)\Big[\bar{H}-1\otimes h_0+(I-\mathfrak{Q})F_c\Big]\bigg]\\
&+\frac{1}{2}trace\bigg[\Psi_0^T\Big(\mathit{\Omega}(k)+\mathit{\Omega}^T(k)\Big)\Psi_0\bigg]\\
\leq{}& 2\big\|\mathit{\Gamma}\mathit{\Omega}(k)\big\|\bigg[\big\|-(\tilde{L}(k)\otimes I_m)(1\otimes f_0)\big\|\\
&+\big\|\mathit{\Upsilon}_1(\eta_t)\dot{\mathit{\Lambda}}_1+\mathit{\Upsilon}_2(\eta_t)\mathit{\Lambda}_1-\mathit{\Upsilon}_3(\eta_t)F_E\big\|+(1-\underline{q}_{min})\omega\big\|\phi\otimes f_0\big\|\\
&+(1-\underline{q}_{min})\omega\big\|-\mathit{\Upsilon}_1(\eta_t)\dot{\mathit{\Lambda}}_1-\mathit{\Upsilon}_2(\eta_t)\mathit{\Lambda}_1+\mathit{\Upsilon}_3(\eta_t)F_E\big\|\bigg]\\
&-2\omega\mathit{\Phi}(1-\bar{q}_{max})\big\|\mathit{\Omega}(k)\mathit{\Gamma}\big\|_1+2g\big\|\mathit{\Omega}(k)\mathit{\Gamma}\big\|\\
&+\frac{1}{2}trace\bigg[\Psi_0^T\Big(\mathit{\Omega}(k)+\mathit{\Omega}^T(k)\Big)\Psi_0\bigg]
\end{aligned}\tag{27}$$

Also, one has

$$\begin{aligned}
&\big\|(\tilde{L}(k)^{-1}\otimes I_m)(\bar{H}-1\otimes h_0)\big\|\leq\big\|\tilde{L}(k)\big\|(\zeta_1\|X_N-X_r\|+\zeta_2\|\dot{X}_N-\dot{X}_r\|)\\
&\leq\big\|\tilde{L}(k)\big\|\big\|\tilde{L}(k)^{-1}\big\|(\zeta_1\Lambda_1+\zeta_2\Lambda_2)\leq\omega(\zeta_1\|\Lambda_1\|+\zeta_2\|\Lambda_2\|)
\end{aligned}\tag{28}$$

Note that, since $\big\|\mathit{\Omega}(k)\mathit{\Gamma}\big\|_1\geq\big\|\mathit{\Omega}(k)\mathit{\Gamma}\big\|$, $V(\mathit{\Gamma},\eta_t,t)=\mathit{\Gamma}^T\mathit{\Omega}(\eta_t)\mathit{\Gamma}=\big\|\Omega(k)^{\frac{1}{2}}\mathit{\Gamma}^T\big\|^2$, and $\big\|\mathit{\Omega}(k)\mathit{\Gamma}\big\|^2=\Big(\Omega(k)^{\frac{1}{2}}\mathit{\Gamma}^T\Big)^T\Omega(k)\Big(\Omega(k)^{\frac{1}{2}}\mathit{\Gamma}^T\Big)\geq\lambda_{min}(\Omega(k))\big\|\Omega(k)^{\frac{1}{2}}\mathit{\Gamma}^T\big\|^2$ then with considering the gain $\mathit{\Phi}$ as (19) and equation (27), one conclude that

$$\mathcal{L}V(\mathit{\Gamma},k,t)\leq-2c_0(k)(\lambda_{min}(\mathit{\Omega}(k)))^{\frac{1}{2}}(V(\mathit{\Gamma},k,t))^{\frac{1}{2}}+\frac{1}{2}trace\bigg[\Psi_0^T\Big(\mathit{\Omega}(k)+\mathit{\Omega}^T(k)\Big)\Psi_0\bigg]\tag{29}$$

equation below depicts generalized Ito formula (Skorohod (1989))

$$\mathbb{E}V(x(\rho_2),\rho_2,r(\rho_2))=\mathbb{E}V(x(\rho_1),\rho_1,r(\rho_1))+\mathbb{E}\int_{\rho_1}^{\rho_2}\mathcal{L}V(x(q),q,r(q))dq\tag{30}$$

using generalized Ito formula (30) and equation (29), one has

$$\frac{d\mathbb{E}[V]}{dt}\leq-2c_0(k)(\lambda_{min}(\mathit{\Omega}(k)))^{\frac{1}{2}}\mathbb{E}[V]^{\frac{1}{2}}+\mathbb{E}\bigg(\frac{1}{2}trace\bigg[\Psi_0^T\Big(\mathit{\Omega}(k)+\mathit{\Omega}^T(k)\Big)\Psi_0\bigg]\bigg)$$

in which $\Psi_1 = \mathbb{E}\left(\frac{1}{2}trace\left[\Psi_0^T\left(\Omega(k) + \Omega^T(k)\right)\Psi_0\right]\right)$, and $\Psi_2 = 2c_0(k)(\lambda_{min}(\Omega(k)))^{\frac{1}{2}}$. Therefore, it follows from the comparison theorem (Michel and Miller (1977)) that

$$\mathbb{E}[V]^{\frac{1}{2}} \leq \frac{\Psi_1}{\Psi_2}\left(W\left(f(\mathbb{E}[V(t_0)]) \times e^{\frac{\Psi_2^2 t}{2\Psi_1}}\right) + 1\right) \tag{31}$$

functions $W(\cdot)$ and $W^{-1}(\cdot)$ denote to Lambert function and its inverse, respectively. one has

$$f(\mathbb{E}[V|_{t_0}]) = W^{-1}\left(\frac{\Psi_2 \mathbb{E}[V|_{t_0}]^{\frac{1}{2}}}{\Psi_1} - 1\right)$$

the Lyapunov function V reaches zero in finite time since the left-hand side of equation (31) is non-negative for $t \leq \frac{2\Psi_1 ln(-(e \times f(\mathbb{E}[V|_{t_0}]))^{-1})}{\Psi_2^2}$ which means the states stochastically converge in mean square sense to mode k of the sliding surfaces in finite-time. $\square$

Remark 3. It should be noted that g is used just to analysis the consensus of the MASs. Indeed, in the distributed controller (13), g can be replaced by $\bar{g} = \bar{\omega}(\bar{\zeta}_1\|\Lambda_1\| + \bar{\zeta}_2\|\Lambda_2\|)$ such that $\bar{\omega}$, $\bar{\zeta}_1$, and $\bar{\zeta}_2$ are chosen large enough to guarantee that the consensus can be reached.

Remark 4. It is worth to mention that in each configuration of the MASs topologies, there is a sliding surface such that when the system jumps from one configuration to another, the sliding surface will also switch from one to another. Therefore, If sliding surfaces can be designed close together sufficiently then chattering also can be reduced by an approximation of the *sign* function while the dynamic of MAS still is completely on a desired sliding surface.

4.5. Sliding Motion

From now on we want to examine stochastic input-to-state stability of the sliding surfaces. Indeed, trajectories of the sliding surfaces converge to desirable band and remain there for all future times.

Definition 4.2. System (9) is said to be a Stochastic Input-to-State Stable (SISS) system, if for any $\varepsilon > 0$, there exist functions $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}$ such that

$$P\{|x(t)| < \beta(|x_0|, t - t_0) + \gamma(\|u\|)\} \geq 1 - \varepsilon, \forall t \geq t_0, \forall x_0 \in \mathbb{R}^n \setminus \{0\} \tag{32}$$

Definition 4.3. For system (9), a function $V(x,t) \in \mathcal{C}^{2,1}(\mathbb{R}^n \times [t_0,\infty);\mathbb{R}_+)$ is called an SISS-Lyapunov function, if there exist functions $\bar{\alpha}, \underline{\alpha} \in \mathcal{K}_\infty$, $\alpha, \chi \in \mathcal{K}$ for all $x \in \mathbb{R}^n, u \in \mathbb{R}^m$, $t \geq t_0$, $\eta_t = k \in \mathcal{S}$, and finite-state measurable Markovian process $\{\eta_t, t \in [0, \mathcal{T}]\}$ such that

$$\underline{\alpha}(|x|) \leq V(x,t,\eta_t) \leq \bar{\alpha}(|x|)$$

and

$$\begin{aligned}\mathcal{L}V(x,t,k) &= \frac{\partial V(x,t,k)}{\partial t} + \frac{\partial V(x,t,k)}{\partial x} f(t,x,k,u) \\ &+ \frac{1}{2} trace\{g^T(t,x,k,u) \frac{\partial^2 V(x,t,k)}{\partial x^2} g(x,t,k,u)\} \\ &+ \sum\nolimits_{j=1}^{\nu} \psi_{kj} V(x,t,j) \leq -\alpha(|x|),\ |x| \geq \chi(\|u\|)\end{aligned}$$

where $\mathcal{L}$ is the infinitesimal generator. SISS-Lyapunov function will be denoted as $(V; \underline{\alpha}, \bar{\alpha}, , \chi)$.

The following theorem explains the connection between SISS-Lyapunov function and the stochastically input-to-state stability of a system in the form of (9).

Theorem 4.4. *The system* (9) *is SISS if there exists an SISS-Lyapunov function* $(V; \underline{\alpha}, \bar{\alpha}, \alpha, \chi)$ *and the function* $\alpha \circ \bar{\alpha}^{-1}$ *is convex.*

Proof. This theorem is an extension of theorem 4.4 in Zhao, Feng, and Kang (2012), thus it is skipped for brevity. □

The next theorem considers the stochastically input-to-state stability of sliding surfaces.

Theorem 4.5. *The sliding surfaces* (9) *are input-to-state stochastically stable, if for the given symmetric positive-definite matrices* $N(k)$*, there exist positive-definite matrices* $P(k)$ *for* $k \in \mathcal{S}$ *satisfy LMI* (44) *or equivalently the equation*

$$2\ P(k)\Xi(k) + \sum\nolimits_{j=1}^{\nu} \psi_{kj} P(j) + N(K) = 0,\ k \in S \tag{33}$$

where $\Xi = \begin{bmatrix} 0 & 1 \\ -\Upsilon_2(k) & -\Upsilon_1(k) \end{bmatrix}$.

Proof. Consider derivative of sliding surfaces (24) which can be written in the following form

$$\begin{bmatrix} \dot{\Lambda}_1 \\ \dot{\Lambda}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\Upsilon_2(k) & -\Upsilon_1(k) \end{bmatrix} \begin{bmatrix} \Lambda_1 \\ \Lambda_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \Upsilon_3(k) \end{bmatrix} F_E + V_w(Y,t) \tag{34}$$

with $Y = \begin{bmatrix} \Lambda_1 \\ \Lambda_2 \end{bmatrix}$.Also, $V_w(Y,t)$ is a random variable with zero mean and appropriate dimension. System (34) can be rewritten as

$$\begin{aligned}\begin{bmatrix} \dot{\Lambda}_1 \\ \dot{\Lambda}_2 \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -\Upsilon_2(k) & -\Upsilon_1(k) \end{bmatrix} \begin{bmatrix} \Lambda_1 \\ \Lambda_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \Upsilon_3(k) \end{bmatrix} F_E + \sigma(Y,t)e(t) \\ &= \Xi(k)Y + \Xi'(k)F_E + \sigma(Y,t)e(t)\end{aligned} \tag{35}$$

which $\sigma(Y,t)$ is variance of white noise $V_w(Y,t)$ and $\{e(t), t \in T\}$ is a sequence of independent equally distributed normal random variables with zero mean and the variance equal to one (normalized white Gaussian noise).

Consider system (34) and the following Lyapunov function,

$$V^{siss}(Y, \eta_t, t) = Y^T P(\eta_t) Y \tag{36}$$

for each possible value $\eta_t = r$, $r \in \mathcal{S}$, weak infinitesimal of the Lyapunov function (36) gives

$$\begin{aligned}
\mathcal{L}V^{siss}(Y, r, t) &= V_t^{siss} + V_Y^{siss} f + trace\Big(\big(\sigma(Y,t)\big)^T P(r)\big(\sigma(Y,t)\big)\Big) + Y^T \sum_{i=1}^{\nu} \psi_{ir} P(r) Y \\
&= 2\ Y^T P(r)\Xi(r) Y + 2\ Y^T P(r)\Xi'(r) F_E \\
&\quad + trace\Big(\big(\sigma(Y,t)\big)^T P(r)\big(\sigma(Y,t)\big)\Big) + Y^T \textstyle\sum_{i=1}^{\nu} \psi_{ir} P(r) Y \\
&= Y^T\Big(2\ P(r)\Xi(r) + \textstyle\sum_{i=1}^{\nu} \psi_{ir} P(r)\Big) Y + 2\ Y^T P(r)\Xi'(r) F_E \\
&\quad + trace\Big(\big(\sigma(Y,t)\big)^T P(r)\big(\sigma(Y,t)\big)\Big)
\end{aligned}$$

considering equation (33) that is $2\ P(r)\Xi(r) + \sum_{i=1}^{\nu} \psi_{ir} P(r) = -N(r)$, one has

$$\begin{aligned}
\mathcal{L}V^{siss}(Y, r, t) &= -Y^T N(r) Y + 2\ Y^T P(r)\Xi'(r) F_E + trace\Big(\big(\sigma(Y,t)\big)^T P(r)\big(\sigma(Y,t)\big)\Big) \\
&= -(1-\theta_*)\big(Y^T N(r) Y\big) + 2\ Y^T P(r)\Xi'(r) F_E - \theta_* Y^T N(r) Y \\
&\quad + trace\Big(\big(\sigma(Y,t)\big)^T P(r)\big(\sigma(Y,t)\big)\Big)
\end{aligned}$$

also by the definition of Frobenius norm

$$trace\Big(\big(\sigma(Y,t)\big)^T P(r)\big(\sigma(Y,t)\big)\Big) = \Big\|\big(\sigma(Y,t)\big)^T L\Big\|_F^2 \tag{37}$$

where L is Cholesky decomposition of $P(r)$, in fact $P(r) = L(r)L^T(r)$, one can conclude that

$$\|Y\|_2 \geq \sqrt{\frac{2\Big\|\Xi'(r)\Big\|\wp_{max}(P(r))\Big\|F_E\Big\|_2 + \Big\|\big(\sigma(Y,t)\big)^T L\Big\|_F^2}{\theta_* \wp_{min}(N(r))}} \tag{38}$$

which can be rewritten as

$$\|Y\|_2 \geq \sqrt{\frac{2\Big\|\Xi'(r)\Big\|\wp_{max}(P(r))\Big\|F_E\Big\|_2 + \sum_{i=1}^{r} \wp_i\Big(\big(\sigma(Y,t)\big)^T L\Big)}{\theta_* \wp_{min}(N(r))}} \tag{39}$$

where $r = \text{rank}\Big(\big(\sigma(Y,t)\big)^T L\Big)$. Therefore, one obtains

$$\mathcal{L}V^{siss}(Y, r, t) \leq -(1-\theta_*) Y^T N(r) Y$$

provided that $|\theta_*| < 1$.

For given positive-definite matrices $N(k)$, $k \in S$, there exist positive-definite matrices $P(k)$, $k \in S$ satisfying

$$2\ P(k)\Xi(k) + \sum\nolimits_{j=1}^{\nu} \psi_{kj} P(j) + N(K) = 0, k \in S \tag{40}$$

which is equivalent to the following inequalities

$$2\ P(k)\Xi(k) + \sum\nolimits_{j=1}^{\nu} \psi_{kj} P(j) \prec 0,\ \ k \in S \tag{41}$$

Pre- and post-multiplying inequality (41) by $P^{-1}(k)$ gives

$$2\ \Xi(k)P^{-1}(k) + P^{-1}(k)\Big(\sum\nolimits_{j=1}^{\nu} \psi_{kj} P(j)\Big)P^{-1}(k) \prec 0 \tag{42}$$

Let $M(k) = P^{-1}(k)$, $k \in S$, then equation (42) yields

$$2\ \Xi(k)M(k) + \psi_{kk}M(k) + M(k)\Big(\sum\nolimits_{j=1, j\neq k}^{\nu} \psi_{kj} P(j)\Big)M(k) \prec 0 \tag{43}$$

Applying Schur complement formula gives LMI (44) where $\Delta(k) = 2\ \Xi(k)M(k) + \psi_{kk}M(k)$, $\forall k \in S$.

$$\begin{bmatrix}
\Delta(k) & \psi_{k1}^{\frac{1}{2}}M(k) & \dots & \psi_{k(k-1)}^{\frac{1}{2}}M(k) & \psi_{k(k+1)}^{\frac{1}{2}}M(k) & \dots & \psi_{k(\nu-1)}^{\frac{1}{2}}M(k) & \psi_{k\nu}^{\frac{1}{2}}M(k) \\
\psi_{k1}^{\frac{1}{2}}M(k) & -M(1) & \dots & 0 & 0 & \dots & 0 & 0 \\
\psi_{k2}^{\frac{1}{2}}M(k) & 0 & \dots & 0 & 0 & \dots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\psi_{k(k-1)}^{\frac{1}{2}}M(k) & 0 & \dots & -M(k-1) & 0 & \dots & 0 & 0 \\
\psi_{k(k+1)}^{\frac{1}{2}}M(k) & 0 & \dots & 0 & -M(k+1) & \dots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\psi_{k(\nu-1)}^{\frac{1}{2}}M(k) & 0 & \dots & 0 & 0 & \dots & -M(\nu-1) & 0 \\
\psi_{k\nu}^{\frac{1}{2}}M(k) & 0 & \dots & 0 & 0 & \dots & 0 & -M(\nu)
\end{bmatrix} \prec 0 \tag{44}$$

□

5. Numerical Example

For our numerical example, we consider the consensus of five 3-Degree of Freedom (DOF) manipulators. The Cartesian space dynamic models are in the form of equations (2) and (3) in which matrices M, C and g and the Jacobian matrix are introduced in (Craig, 2005). Figure 1 shows a 3-DOF nonplanar manipulator. Let mass of each link to be $m_1 = 0.3\ Kg$, $m_2 = 0.25\ Kg$ and $m_3 = 0.15\ Kg$. We consider each link to be a rectangular solid of homogeneous density with dimensions l_i, d_i, and h_i where $l_1 = 0.25\ m$, $l_2 = 0.15\ m$, $l_3 = 0.1\ m$, and $h_i = d_i = 0.02\ m$, $i \in \{1, \cdots, m\}$. In addition, The resulting inertia tensor written in the frame at the center of mass is are

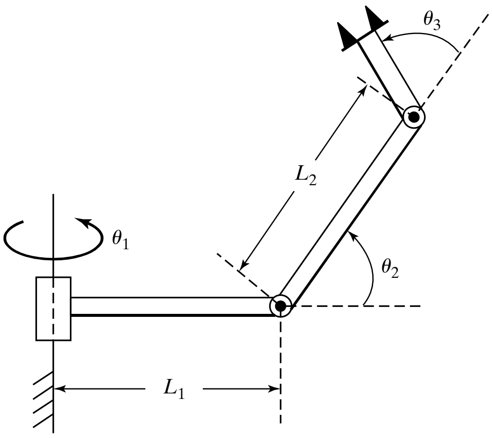
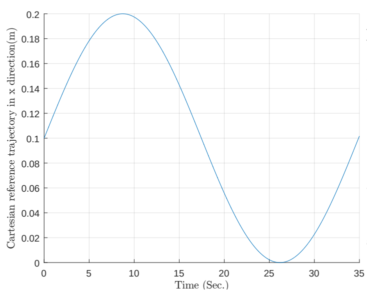


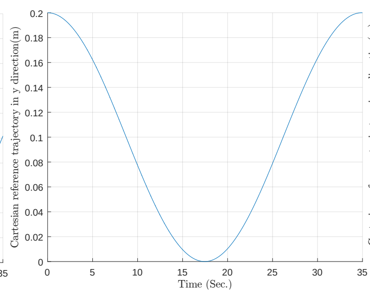
Figure 1. The 3R nonplanar arm (Craig, 2005).

defined as ${}^{C_i}I_i = \text{diag}\Big(I_{xi}, I_{y_i}, I_{zi}\Big)$ in which $I_{xi} = \frac{m_i}{12}(h_i^2 + l_i^2)$, $I_{y_i} = \frac{m_i}{12}(d_i^2 + h_i^2)$, and $I_{zi} = \frac{m_i}{12}(d_i^2 + l_i^2)$, $i \in \{1, \cdots, m\}$. Also, the centre of mass for each link are ${}^iP_{c_i} = [\frac{l_i}{2}, 0, 0]^T$. The acceleration of gravity is considered to be $9/8\ \frac{m}{s^2}$.

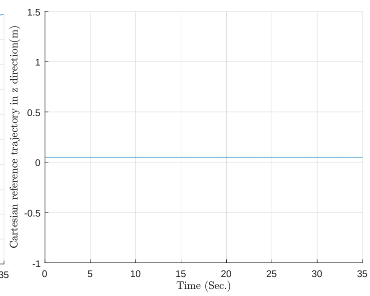
$\bar{F}$ and $\bar{F}_E$ are the control and the environment force vector applied to the end-effectors, respectively. The reference trajectory in Cartesian space is a circle with radius 10 cm and constant attitude $z = 5\ cm$. Cartesian reference trajectories in x, y, and z directions are shown in Figure 2. Switching signal of communication topologies



(a) x direction.



(b) y direction.



(c) z direction.

Figure 2. Cartesian reference trajectory.

resulted from Markovian jump structures is depicted in Figure 3. Also, Figure 4 shows the interaction topologies of the manipulators. We assume that there is a spanning tree in all topologies which means that Assumption 2.2 is satisfied. In addition, a transition probability matrix is considered as follow

$$\begin{bmatrix} 0.0251 & 0.2247 & 0.1301 & 0.0516 & 0.1613 & 0.2110 & 0.1566 & 0.0396 \\ 0.0858 & 0.0833 & 0.1439 & 0.2632 & 0.1221 & 0.1354 & 0.1585 & 0.0078 \\ 0.1206 & 0.1295 & 0.1536 & 0.1236 & 0.1779 & 0.0644 & 0.1418 & 0.1186 \\ 0.0400 & 0.0062 & 0.2267 & 0.1124 & 0.2027 & 0.1937 & 0.0909 & 0.1275 \\ 0.1178 & 0.2247 & 0.0534 & 0.1454 & 0.1338 & 0.0295 & 0.0295 & 0.2660 \\ 0.1260 & 0.0272 & 0.3116 & 0.0024 & 0.0512 & 0.2651 & 0.0719 & 0.1445 \\ 0.0349 & 0.0169 & 0.0497 & 0.1865 & 0.1990 & 0.2460 & 0.2587 & 0.0087 \\ 0.2049 & 0.0869 & 0.0225 & 0.0640 & 0.1413 & 0.0950 & 0.1732 & 0.2122 \end{bmatrix} \tag{45}$$

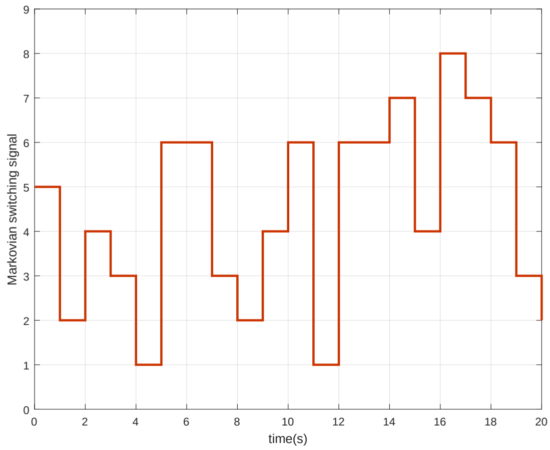
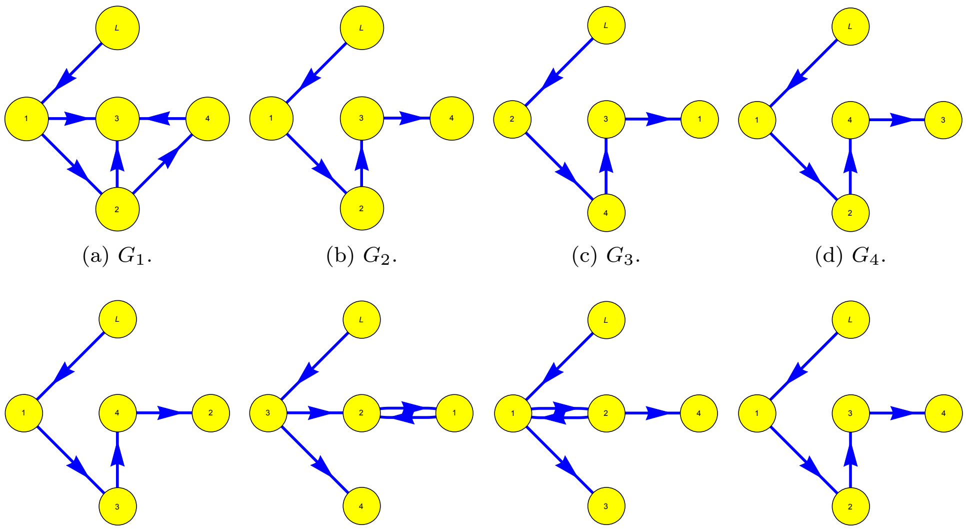


Figure 3. Markovian Switching Signal.

Let the intensity of the measurement noise to be $\sigma_{ji} = 0.001,\ \forall i, j = \{1, \cdots, N\}$. The initial states of agents' positions and velocities are $x_1(0) = [0.2, 0.25, 0.07]^T$, $x_2(0) = [0.05, 0.05, 0.06]^T$, $x_3(0) = [0.16, 0.3, 0.04]^T$, $x_4(0) = [0.15, 0.05, 0.03]^T$ and $V_1(0) = V_2(0) = V_3(0) = V_4(0) = [0, 0, 0]^T$. In this simulation, the manipulators are



(a) G_1. (b) G_2. (c) G_3. (d) G_4.

(e) G_5. (f) G_6. (g) G_7. (h) G_8.

Figure 4. Communication topologies.

impedance controlled to regulate the environment force (or disturbance) $\bar{F}_E$ when the manipulator interacts with the environment or human operator to follow the desired path closely in both free and constrained motions.

Due to page limit, simulation results just for the follower 2 are shown here.

The follower 2 interacts with its environment while a Cartesian linear force $\bar{f}_{2e}^T$ as a disturbance is acting on its end-effector in y direction (constrained motion) between the time $t = 10s$ to $t = 12s$ which is depicted in Figure 5. Also, the environment force before and after the contact was zero except during the collision. This force may be described as $\bar{f}_{2e}^T = [0, 0.02\sin(5t), 0]^T$. According to Assumption 3.2, it should be noted that $\bar{f}_{2e}^T$ is locally bounded. In addition, the proposed distributed controllers (11) to

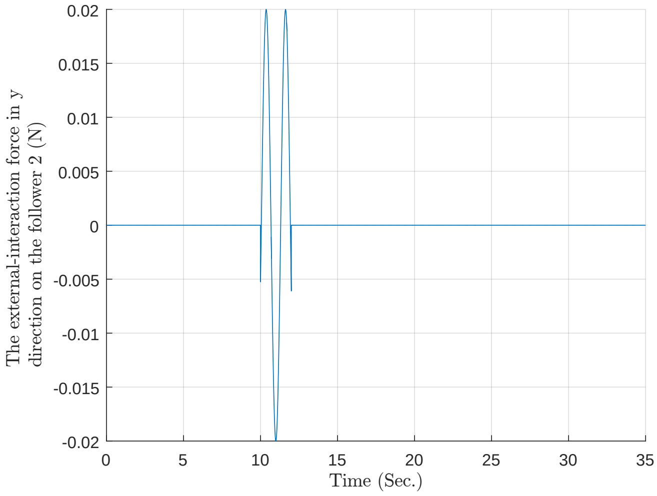


Figure 5. The environment force.

(13) and the desired sliding surfaces (9) are used in the simulation with the following parameters $\Upsilon_1(k) = M_d^{-1}C_d$, $\Upsilon_2(k) = M_d^{-1}K_d$, and $\Upsilon_3(k) = M_d^{-1}$ with $M_d = 0.2 \times I_3$, $K_d = I_3$, and $C_d = 1.7 \times I_3$, for any $k \in \mathcal{S}$. Also, we consider $\xi_{ii}(k) = 1$ for any $k \in \mathcal{S}$ and $i \in \{1, \cdots, N\}$. Therefore, one obtains $\Xi(k) = I_5$, $\forall k \in \mathcal{S}$.

Consider the transition probability matrix (45) and ρ_{ii}, $i \in \{1, \cdots, N\}$ then $\Omega(k)$ for any $k \in \mathcal{S}$ are given as $\Omega(1) = \Omega(2) = \Omega(3) = \Omega(4) = \Omega(5) = \Omega(6) = \Omega(7) = \Omega(8) = 10.1103 \times I_5$. Therefore, the LMI (10) for any $k \in \mathcal{S}$ and $0 \leq \underline{q}_{min} < 1$, $i = \{1, 2, \cdots, N\}$ and $j = \{1, 2, \cdots, m\}$ is feasible.

Now we verify the feasibility of equation (33) or equivalently LMI (44). Consider Theorem 4.5 in which

$$\Xi(k) = \begin{bmatrix} 0 & 1 \\ -\Upsilon_2(k) & -\Upsilon_1(k) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -8.5 & -1 \end{bmatrix}, \ \forall k \in \mathcal{S}.$$

For each topology, we compute the positive definite matrix P through equation (33) as follow.

$$P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = P(7) = P(8) = \begin{bmatrix} 23.3945 & 1.3761 \\ 1.3761 & 2.7523 \end{bmatrix}.$$

5.1. Simulation Results: Fault-free Case

Now we assume that there is no fault in the actuators. In other words, $q_{ij} = 0$, $i = \{1, 2, \cdots, N\}$ and $j = \{1, 2, \cdots, m\}$. Figure 6 shows that the end-effectors track the Cartesian space reference trajectory and it shows that the followers achieved the consensus on the desired trajectory. Cartesian trajectory tracking of the follower 2 and the corresponding errors in x, y, and z directions are shown in Figure 7. The sliding surfaces, the control force applied to the end-effector and torques applied to the joints of the follower 2 are depicted in Figure 8 and Figure 9.

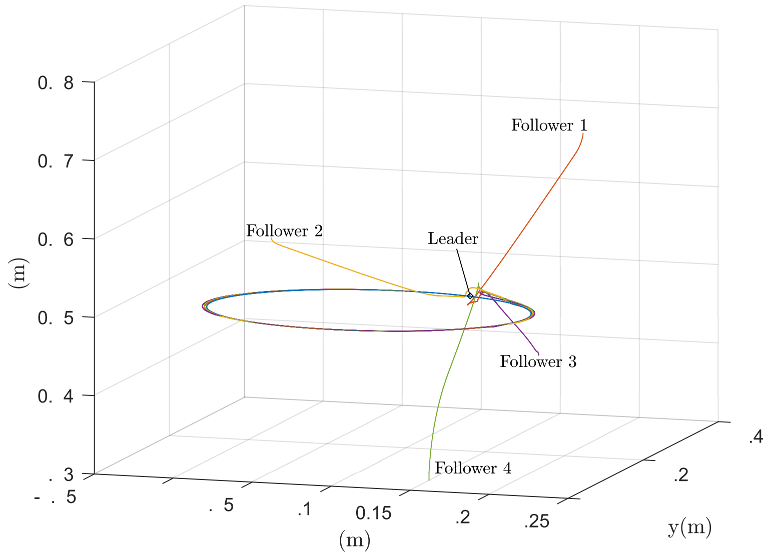


Figure 6. Trajectory tracking for all agents.

5.2. Simulation Results: Faulty Case

It is assumed that at $t = 5s$, the actuator of the joint 1 in the follower 3, the actuator of the joint 2 in the manipulator 4 and the actuator of the joint 2 in the follower 2 are faulty with the effectiveness loss values $q_{31} = 0.5$, $q_{42} = 0.6$, and $q_{22} = 0.611$ and other actuators are still normal ($q_{ij} = 0$, for $i = \{1, 2, \cdots, N\}$ and $j = \{1, 2, \cdots, m\}$). Also, fault model is given as (8) with $\underline{q}_{ij} = 0$ and $\bar{q}_{ij} = 0.9$ for $i = \{1, 2, \cdots, N\}$ and $j = \{1, 2, \cdots, m\}$. The simulation results without and with applying fault-tolerant control are shown in Figure 10-13 and Figure 14-17, respectively. Simulation results show that the proposed passive fault tolerant control approach ensures the acceptable performance of the closed-loop system.

6. Conclusion

In this paper, the focus of attention was on fault-tolerant control for the consensus of nonlinear MASs with directed link failures/recoveries in the presence of communication noise and actuator faults. We have addressed a definition for stochastic input-to-state stable Lyapunov function and a theorem for assurance of this type of stability based on SMC method and weak infinitesimal operation in the terms of LMIs. Furthermore, randomly switching topologies in MASs governed by Markovian jump process. Simulations have been given to illustrate validity of the presented algorithms for the nonlinear Lagrangian MASs. In the terms of future work, network-induced time delays and adaptive estimation of the effectiveness loss values in the fault model can be considered.

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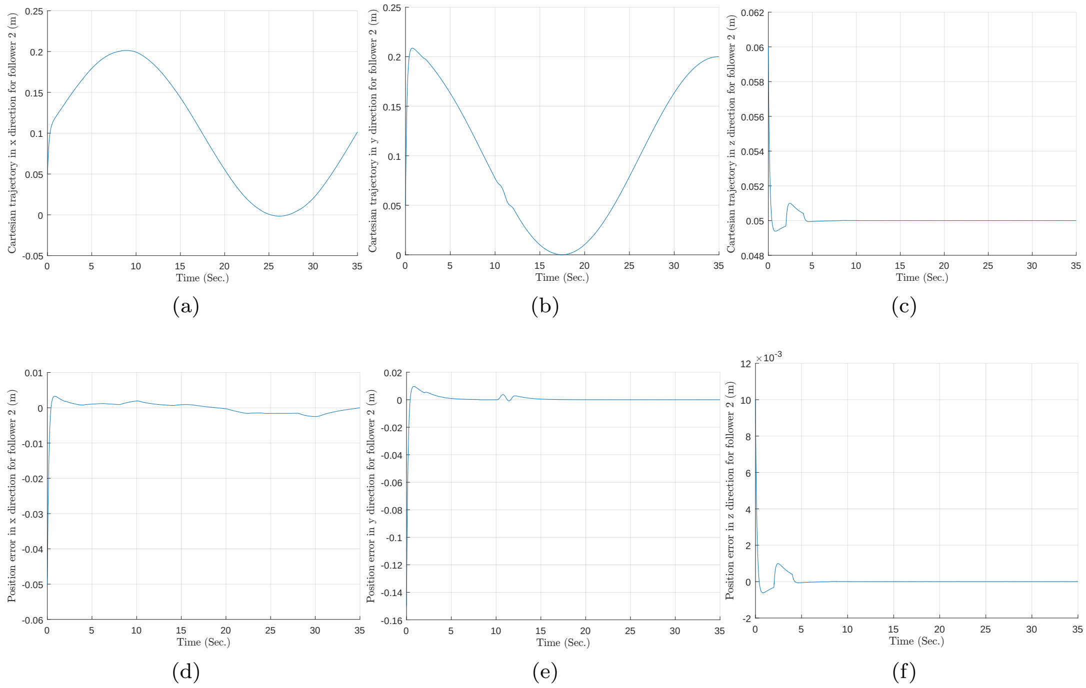

Figure 7. Cartesian trajectory tracking of follower 2 in (a):x direction. (b):y direction. (c):z direction. and corresponding errors in (d):x direction. (e):y direction. (f):z direction.

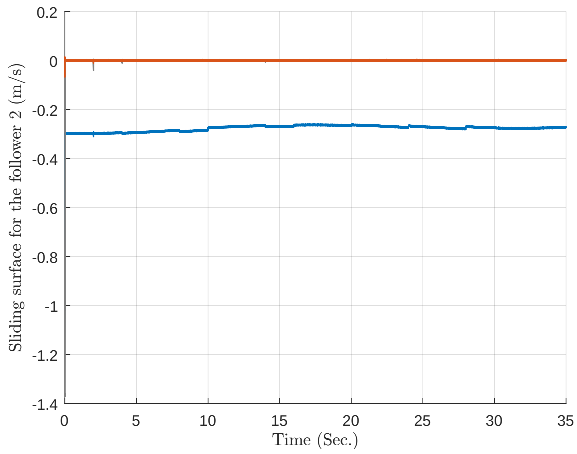

Figure 8. The stochastic sliding surface for follower 2.

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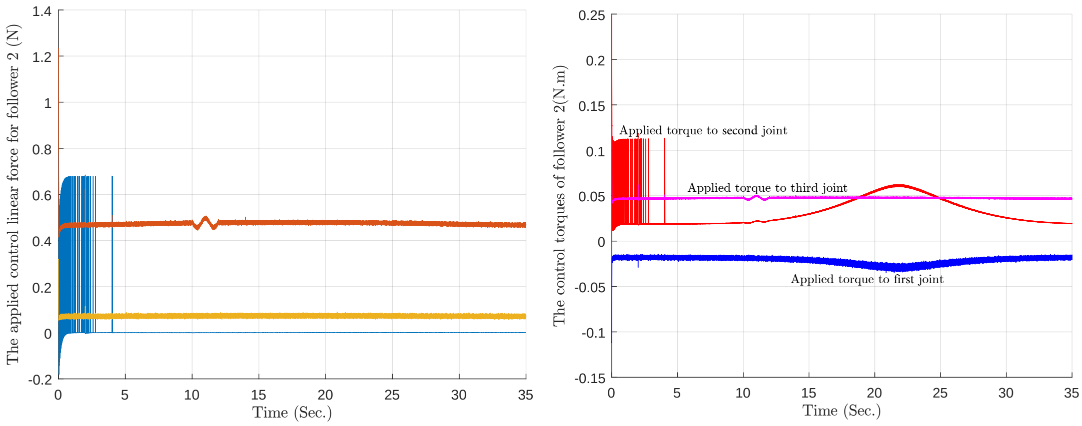

(a) (b)

Figure 9. (a): The control linear forces applied to the end-effector of follower 2. (b):The control torques applied to the joints of follower 2

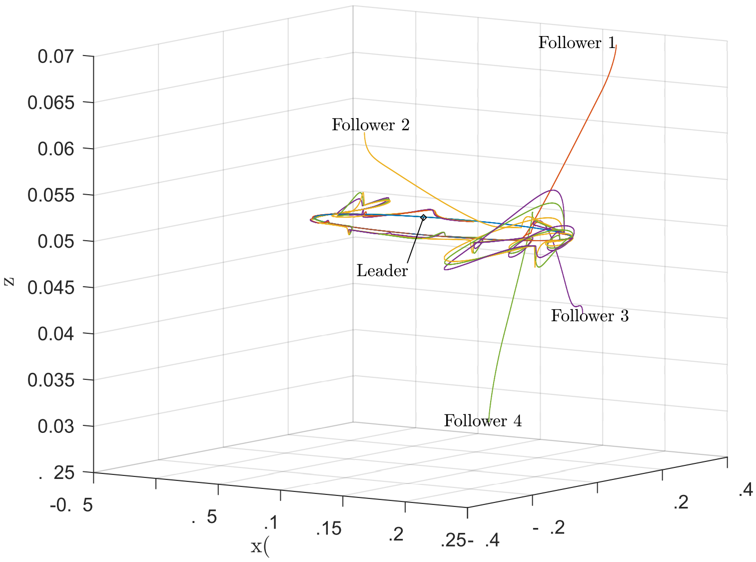

Figure 10. Trajectory tracking for all agents in the presence of the actuator fault without the fault-tolerant control.

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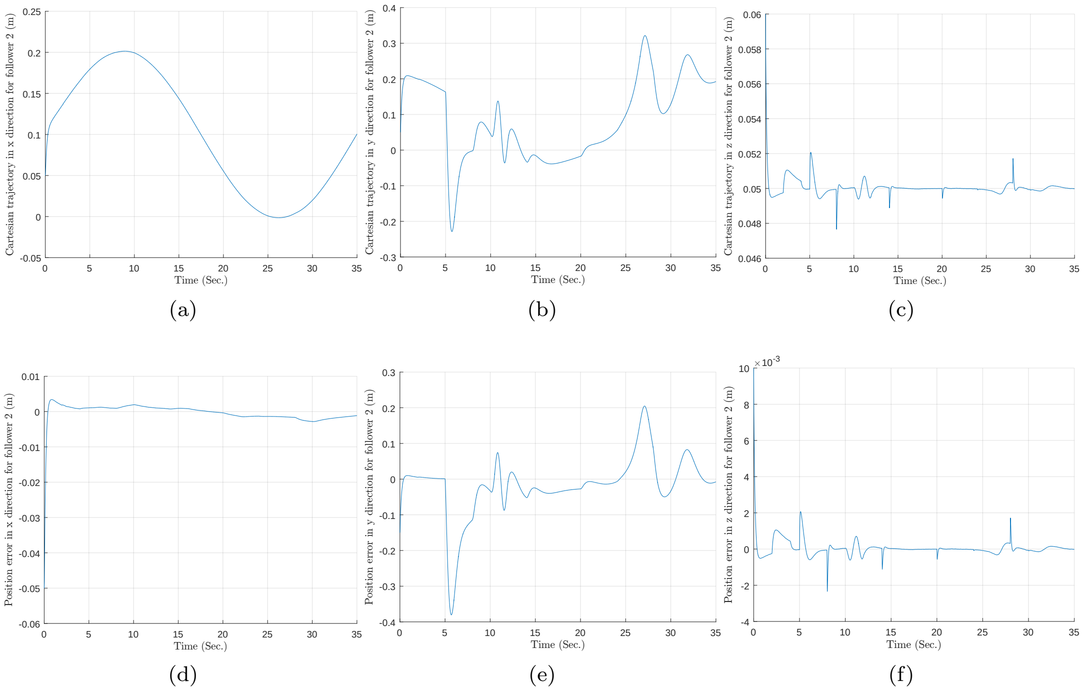

Figure 11. Cartesian trajectory tracking of follower 2 in (a):x direction. (b):y direction. (c):z direction. and Corresponding errors in (d):x direction. (e):y direction. (f):z direction, in the faulty case and without the fault-tolerant control

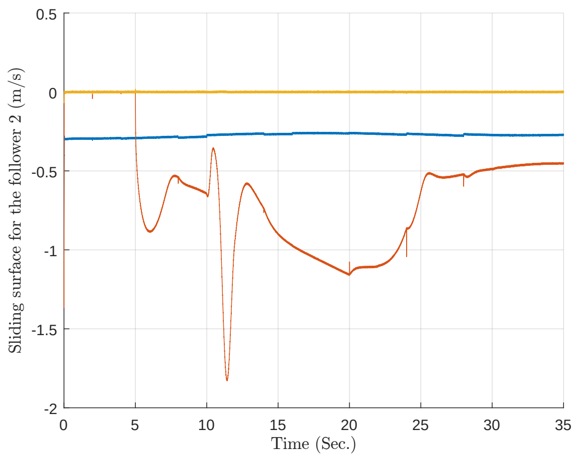

Figure 12. The stochastic sliding surface for follower 2: the faulty case and without the fault-tolerant control.

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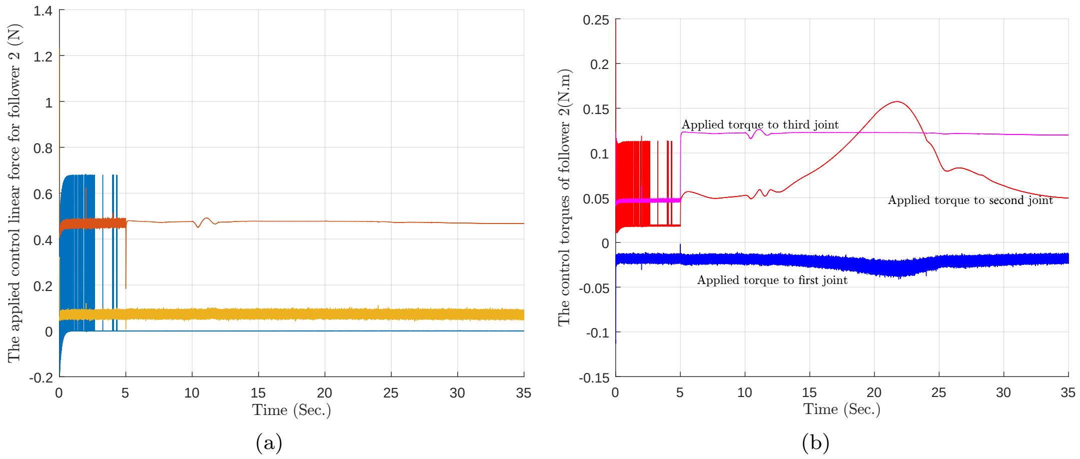

Figure 13. (a): The control forces applied to the end-effector of follower 2. (b):The control torques applied to the joints of follower 2: the faulty case and without the fault-tolerant control

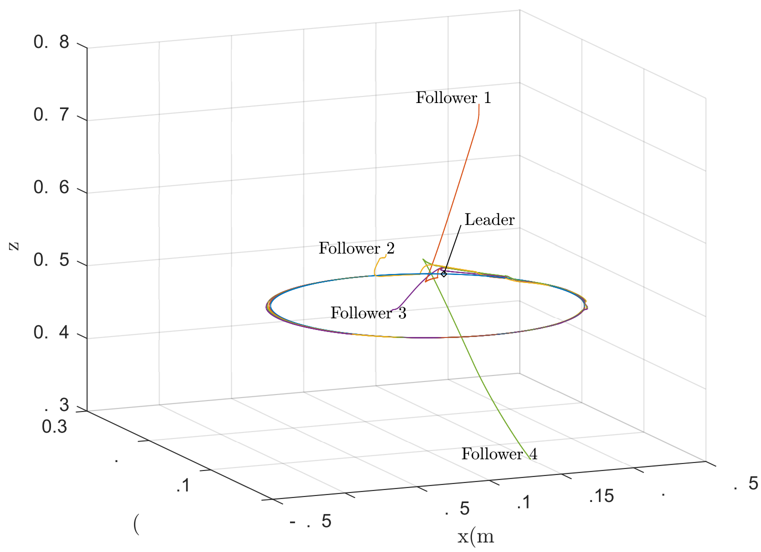

Figure 14. Trajectory tracking for all agents in the presence of the actuator fault with the fault-tolerant control.

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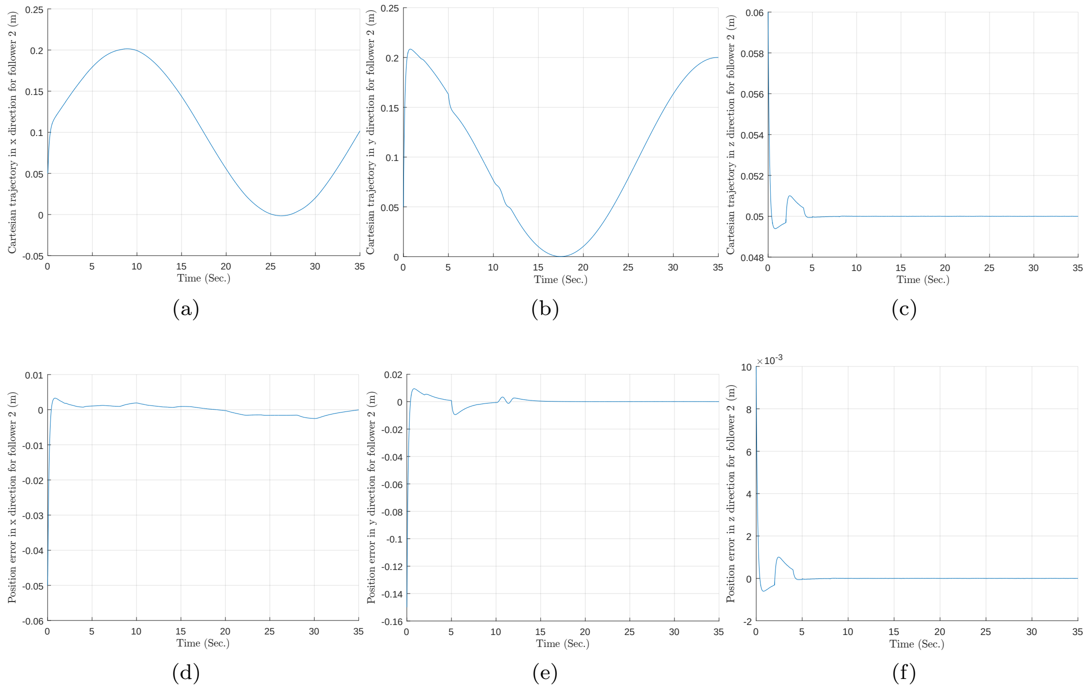

Figure 15. Cartesian trajectory tracking of follower 2 in (a):x direction. (b):y direction. (c):z direction. and Corresponding errors in (d):x direction. (e):y direction. (f):z direction. the faulty case and with the fault-tolerant control

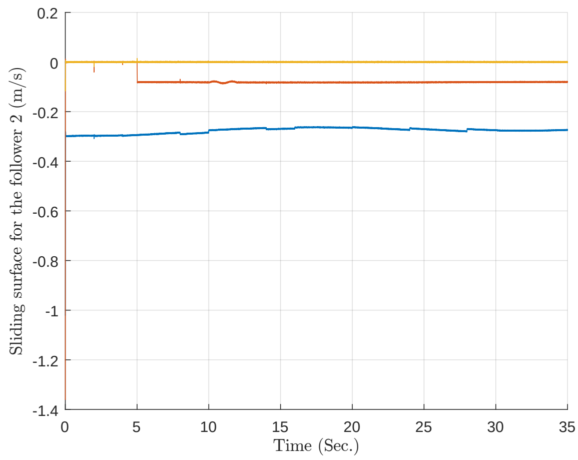

Figure 16. Stochastic sliding surface for follower 2: the faulty case and with the fault-tolerant control.

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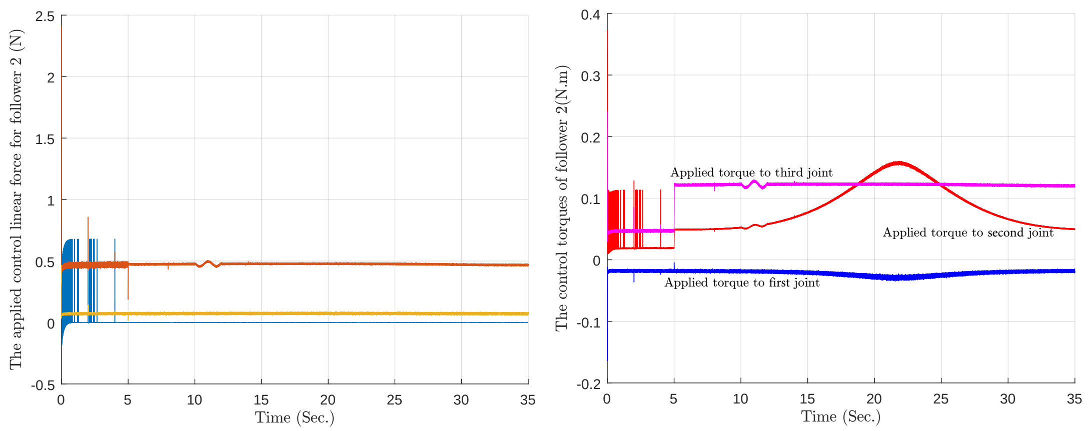

(a) (b)

Figure 17. (a): The control forces applied to the end-effector of follower 2. (b):The control torques applied to the joints of follower 2: the faulty case and with the fault-tolerant control

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